Unconventional Fiscal Policy in HANK

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First Version: March, 2020
This Version: July 5, 2023

Abstract

We show that in a New Keynesian model with household heterogeneity, fiscal policy can be a perfect substitute for monetary policy: three simple conditions for consumption taxes, labor taxes, and the government debt level are sufficient to induce the same consumption and labor supply of each household and, thus, the same allocation as interest rate policies. When monetary policy is constrained by a binding lower bound, a currency union, or an exchange rate peg, fiscal policy can therefore replicate any allocation that hypothetically unconstrained monetary policy would generate.

Keywords: Unconventional Fiscal Policy, Heterogeneous Agents, Incomplete Markets, Liquidity Trap, Sticky Prices

JEL Codes: E12, E21, E24, E43, E52

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1 Introduction

Monetary policy oftentimes cannot freely adjust the nominal interest rate—be it due to a binding lower bound, a currency union, or an exchange rate peg. In these environments of constrained monetary policy, policymakers need to resort to alternative stabilization tools. Recent real-world episodes suggest that unconventional fiscal policy tools such as changes in the consumption tax rates are promising alternatives to conventional interest rate policies as they stimulate consumption through the intertemporal substitution channel (Bachmann et al. (2021), Baker et al. (2019), D'Acunto et al. (2022)).

In this paper, we show that a mix of unconventional fiscal policy tools can be a perfect substitute to monetary policy in a heterogeneous agent New Keynesian (HANK) model. In particular, we show that three simple conditions for consumption taxes, labor taxes, and the government debt level are sufficient to generate the same consumption and labor supply of *each* household and, thus, the same allocation as monetary policy. This perfect substitutability result holds when monetary policy is constrained meaning that our unconventional fiscal policy measure—which we label *HANK unconventional fiscal policy (HANK-UFP)*—circumvents the constraints of monetary policy. In particular, we show at the Effective Lower Bound (ELB) that HANK-UFP can generate any allocation that hypothetically unconstrained monetary policy could achieve by inducing the same cross-sectional consumption and labor supply.

The intuition for our perfect substitutability result is that HANK-UFP affects the optimization problem of each household in the same way as a change in the interest rate thereby replicating its whole transmission mechanism: HANK-UFP and interest rate changes induce the same inter- and intratemporal incentives for consumption and labor supply as well as the same effects on each household's budget constraint. In that sense, our analysis builds on the perfect substitutability result between fiscal and monetary policy in a representative agent New Keynesian (RANK) model (see Correia et al. (2008), Correia et al. (2013)). In RANK, consumption taxes and labor taxes alone—*unconventional fiscal policy (UFP)*—are sufficient to induce the same optimization problem of the representative household as monetary policy since they induce the same inter- and intratemporal incentives for consumption and labor supply. However, this result relies on the fact that, by construction, policies do not redistribute across households as all of the income accrues to the same household. In contrast, in HANK models, households are heterogeneous both in their income and in their income compositions. One of our contributions is to show that, as a consequence, tax policies alone are no longer sufficient to induce the same optimization problem of each household in HANK. The reason is that tax policies alone have different effects on the various income components of households and, hence, affect households' budget constraints differently than interest rate policies. In addition, these different cross-sectional effects induce different aggregate effects since households are heterogeneous in their marginal propensities to consume (MPCs). We show that, as a consequence, tax policies alone are not able to stabilize the economy while the ELB is binding. Furthermore, tax policies alone push the economy to a new steady state after the ELB stops binding characterized by lower real interest rates and higher inefficiencies out of incomplete markets compared to the original steady state.

For our analysis, we extend the textbook New Keynesian model by a standard heterogeneous agent, incomplete markets set-up. We assume that households face uninsurable idiosyncratic income risk and borrowing constraints. Households self-insure against idiosyncratic shocks to their labor productivity by buying risk-free bonds. Monetary policy sets the interest rate and fiscal policy sets proportional taxes on consumption and on labor, issues government debt, and pays lump-sum transfers to households. We assume that households have perfect foresight. In this environment, we analytically characterize our fiscal policy scheme, HANK-UFP, which induces the same optimization problem of each household as a change in the interest rate. This is sufficient to generate the same effects through general equilibrium since monetary policy and fiscal policy do not affect firms' equilibrium conditions directly but only through the household block.

To fix ideas, consider the effects of expansionary monetary policy on the households' optimization problems which are replicated by HANK-UFP as follows. In line with Correia et al. (2013), tax policies generate the same inter- and intramteporal incentives as monetary policy: pre-announced paths for higher future consumption taxes trigger the same incentives to intertemporally substitute consumption as a decrease in the real interest rate. Yet, higher consumption taxes also incentivize households to reduce their labor supply for any given real wage. Lower labor taxes offset this effect of consumption taxes on the labor supply.

We prove that when these tax policies are combined with debt policies in the form of an increase in the government debt level, each household's income and therefore her budget constraint is identically affected by HANK-UFP and monetary policy. We show that for this to be the case, it is sufficient that monetary policy and HANK-UFP induce the same redistribution through the policy block.[1] Expansionary monetary policy redistributes from asset holders to the government: on the one hand, lower interest rates induce a negative

[1] Among others, Bhandari et al. (2021), Bilbiie (2018), and Acharya and Dogra (2020) highlight the effects of households' heterogeneous exposure to a policy change arising indirectly through changes in output. While households are also heterogeneously exposed to changes in output in our model, this does not affect our perfect substitutability result because these effects are the same with HANK-UFP and monetary policy since both identically affect output. Thus, for our analysis, it is sufficient to focus on the heterogeneous exposure of households to policy changes arising directly from changes in monetary and fiscal policy variables, that is, through the policy block.

wealth effect which affects households in proportion to their asset holdings. On the other hand, the government issues the assets and, hence, has lower interest rate payments which shifts resources to the government. These additional resources are then redistributed back to households through a fiscal response. HANK-UFP replicates this redistribution through the policy block as follows. Higher consumption taxes generate the same negative wealth effect on the assets of households as they decrease the consumption value of assets. This again hurts households in proportion to their asset holdings. As households accumulate these assets for self-insurance purposes, higher consumption taxes increase the precautionary savings demand of households in proportion to their asset holdings. The government accommodates this higher asset demand by increasing the government debt level such that the value of total assets in consumption value terms is the same as in the monetary policy case. This provides the government with the same additional resources as in the monetary policy case which triggers the same fiscal response and, thus, the same redistribution back to households.

Our perfect substitutability result between fiscal policy and monetary policy is especially relevant when conventional monetary policy is constrained. We therefore apply our perfect substitutability result at the ELB—a typical case of constrained monetary policy—and show that HANK-UFP circumvents the constraint. By increasing consumption taxes, decreasing labor taxes, and permanently increasing the government debt level, HANK-UFP replicates the allocation associated with hypothetically unconstrained monetary policy—the counterfactual in which monetary policy could freely set nominal interest rates without any lower bound constraints.

In this ELB environment, we quantify the role of debt policies—the novel instrument that is necessary for perfect substitutability in HANK. To this end, we study through the lens of our HANK model the UFP scheme of Correia et al. (2013) which only consists of consumption taxes and labor taxes replicating the inter- and intratemporal incentives of households. Since government debt is now not adjusted for the consumption value but the same as in the unconstrained monetary policy case, the fiscal response that monetary policy induces through reducing the interest rate payments cannot be replicated by this fiscal policy scheme. As a consequence, this tax policy scheme cannot fully stabilize the economy in the short-run. We further show that with this tax-only policy scheme the economy now converges to a new steady state in which the real interest rate is lower than in the original steady state. The reason is that fiscal policy cannot satisfy the higher precautionary savings demand of households in the long-run induced by the permanently higher consumption taxes. Consequently, households are worse insured against their idiosyncratic income risk which permanently increases the inefficiency from incomplete markets. We show that as a consequence, each household's welfare is lower compared to the HANK-UFP scenario. This shows that tax policies which are neutral in RANK in the long-run can induce long-run inefficiencies in HANK.

While our baseline model abstracts from sticky wages and capital, we show analytically that fiscal policy can also be a perfect substitute for monetary policy if we extend our model by these model features. We first include sticky wages implemented via unions into the model and show that the same three conditions for consumption taxes, labor taxes, and the government debt level are sufficient for perfect substitutability. The reason is that given HANK-UFP, the unions face the same wage-setting problem as in the monetary policy case since HANK-UFP replicates the consumption of each household. When we add capital to our model, monetary policy affects the return on capital which changes the incentives to use capital for firms and which affects households' budget constraints. We show that HANK-UFP can still be a perfect substitute for monetary policy if it is extended. A condition for capital subsidies paid to the firms allows HANK-UFP to replicate firms' incentives to use capital in the spirit of Correia et al. (2013). How HANK-UFP replicates the effects on households' budget constraints depends on the degree of substitutability of bonds and capital: if they are perfect substitutes, the effects on the budget constraints are replicated by an extra increase in the government debt level. If capital is illiquid and, hence, an imperfect substitute for bonds, the government needs to issue an additional asset which has the same pay-off structure as capital. In both cases, the intuition from our baseline result carries over in the sense that an increase in consumption taxes decreases the consumption value of capital holdings. Accordingly, the extra bonds or, in the illiquid capital case, the additional assets are held by households in proportion to their capital holdings. In both cases, these extra asset holdings, in turn, finance the capital subsidies. This way, HANK-UFP redistributes from capital-rich households to firms in the same way as lower interest rates redistribute from capital-rich households to firms and, thus, the effects on each household's budget constraint are the same.

Related literature. Feldstein (2002) and Hall (2011) propose to increase future consumption taxes when monetary policy is constrained by the ELB. Baker et al. (2019), Bachmann et al. (2021), and D'Acunto et al. (2022) empirically show that recent real-world episodes of consumption tax policies have stimulated consumption and, thus, aggregate output through the intertemporal substitution channel. We show how these consumption tax policies can be part of a larger fiscal mix which does not only replicate the intertemporal substitution channel but the whole transmission channel of monetary policy.

Correia et al. (2008) and Correia et al. (2013) show that a combination of consumption taxes and labor taxes is a perfect substitute for monetary policy in RANK by replicating its effects on the policy wedges in the household's first-order conditions. Bianchi-Vimercati et al. (2021) show that in RANK, the effectiveness of these tax policies is not affected by bounded rationality in the form of level-k thinking. We depart from the textbook RANK model in a different way than Bianchi-Vimercati et al. (2021) and show that Correia et al. (2013)'s seminal result relies on the fact that monetary and fiscal policy do not redistribute among households in RANK. We show that when households are heterogeneous in their income composition, UFP as prescribed by Correia et al. (2013) is no longer a perfect substitute for monetary policy. We further show that in HANK, tax policies alone cannot fully stabilize the economy in the short-run and, in addition, push the economy to a new steady state characterized by a lower real interest rate.

Our analysis is also related to a large literature on the transmission mechanism of monetary policy in HANK (see among many others Werning (2016), McKay et al. (2016), Kaplan et al. (2018), Bilbiie (2018), Auclert (2019), Hagedorn et al. (2019a), Acharya and Dogra (2020), Auclert et al. (2020), Luetticke (2021)).[2] Recently, the HANK literature has also studied fiscal policy. Auclert et al. (2018), Ferriere and Navarro (2018), and Hagedorn et al. (2019b) analyze fiscal multipliers in HANK models. Unlike our paper, these papers do not study whether fiscal policy can replicate the allocations of unconstrained monetary policy. Oh and Reis (2012) and Bayer et al. (2020) show that transfer policies can have large effects in HANK models which is also reflected in our numerical analysis. As in our analysis, Bayer et al. (2023) show that the government debt level affects the economy in the short- and in the long-run. In particular, they show how the government debt level affects the liquidity spread which, in turn, affects the real economy. In contrast, we provide a specific rule for the government debt level as part of a set of sufficient conditions such that fiscal policy is a perfect substitute for monetary policy. Bhandari et al. (2021) analyze optimal fiscal and monetary policy in a HANK model with aggregate risk. They analyze how idiosyncratic insurance possibilities shape optimal monetary and fiscal policy in the face of aggregate shocks while we focus on the substitutability of monetary policy by fiscal policy. Unlike our analysis, Bhandari et al. (2021) include aggregate risk, but they abstract from borrowing constraints, a binding ELB, and consumption taxes. Le Grand et al. (2021) also study optimal fiscal and monetary policy in a HANK model. Their set of fiscal instruments can replicate the allocations of monetary policy away from the ELB in their HANK model. Unlike our HANK-UFP scheme, they use capital taxes instead of consumption taxes. This is an important distinction from our analysis for two reasons: first, capital taxes face the

[2] There is also a growing literature analyzing the transmission mechanism of monetary policy in models with firm heterogeneity, see among others Reiter et al. (2013), Koby and Wolf (2020), and Ottonello and Winberry (2020).

same limitations as interest rate policies since the nominal after-tax return on savings cannot become negative. Hence, capital taxes cannot be used to circumvent the ELB constraint. Second, capital taxes have different effects on the budget constraints of households compared to consumption taxes.

Wolf (2021) is closest to our paper. He shows that transfer policies can achieve the same aggregate outcomes as interest rate cuts in a linearized HANK model with sticky wages. Unlike our perfect substitutability result, Wolf (2021) derives his aggregate equivalence result by showing that lump-sum transfers can trigger the same *aggregate* consumption response in partial equilibrium as monetary policy. In a linearized environment with sticky wages in which labor unions consider the marginal utility of aggregate consumption when setting wages, this then generates the same responses of macroeconomic aggregates through general equilibrium. Unlike Wolf (2021), we show that with tax and debt policies, fiscal policy can directly manipulate the optimization problem of each household in the same way as monetary policy. Thus, our result differs from the result in Wolf (2021) in two aspects: first, HANK-UFP does not only achieve equivalence in aggregates but also in the cross-section of households and, thus, the distribution of households evolves in the same way with monetary and fiscal policy. Second, this implies that our result also holds if households' labor supply is affected by their individual consumption in the short-run as the cross-sectional consumption and labor supply is the same as with monetary policy. To the best of our knowledge, we are the first who show how fiscal policy can circumvent constraints of monetary policy, including the ELB constraint, in a HANK model.

Outline. Section 2 presents our HANK model. Section 3 shows analytically that HANK-UFP is a perfect substitute for monetary policy. Section 4 provides a numerical analysis to show how HANK-UFP circumvents the ELB constraint and highlights the role of debt policies in HANK-UFP. Section 5 concludes.

2 Model

This section outlines our HANK model which is a sticky-price New Keynesian model extended by a standard heterogeneous households, incomplete markets set-up.

2.1 Households

The economy is populated by a continuum of households who are identical in their preferences given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_{h,t}^{1-\gamma}}{1-\gamma} - \frac{l_{h,t}^{1+\psi}}{1+\psi} \right], \tag{1}$$

where β denotes the household's discount factor, $c_{h,t}$ denotes consumption of household h in period t, and $l_{h,t}$ denotes her labor supply. The parameters γ and ψ govern the degree of risk aversion and the inverse Frisch elasticity, respectively.

The budget constraint of household h and her borrowing constraint are given by:

$$(1+\tau_t^C)c_{h,t} + b_{h,t+1} = (1+r_t)b_{h,t} + (1-\tau_t^L)w_t z_{h,t} l_{h,t} + D_t + Tr_t$$
$$b_{h,t+1} \geq 0,$$

where $c_{h,t}$ denotes consumption of household h, $b_{h,t}$ are 1-period risk-free bonds which are issued by the government. r_t is the real interest rate paid on these bonds between period $t-1$ and period t. In addition, there is a proportional tax rate on consumption, τ_t^C, and a proportional tax rate on her individual labor income, τ_t^L. The labor income consists of the wage rate, w_t[3], the individual productivity level, z_t, and the individual labor supply. Since $z_{h,t}$ evolves according to an exogenous finite-state Markov chain, households face idiosyncratic income risk. As in McKay et al. (2016), we assume that all households receive an equal share of firms' dividends, D_t[4], and a lump-sum transfer, Tr_t, from the government. For our analytical analysis in Section 3, it is useful to represent the budget constraint in consumption value terms:

$$c_{h,t} + \frac{b_{h,t+1}}{1+\tau_t^C} = (1+r_t)\frac{b_{h,t}}{1+\tau_t^C} + \frac{1-\tau_t^L}{1+\tau_t^C} w_t z_{h,t} l_{h,t} + \frac{D_t + Tr_t}{1+\tau_t^C} \tag{2}$$
$$b_{h,t+1} \geq 0.$$

We assume the standard Bewley-Huggett-Aiyagari incomplete markets setup such that there are no state-contingent securities. As households cannot buy perfect insurance, they accumulate government bonds to self-insure their idiosyncratic risk. As a consequence, households differ in their individual states which consists of household's asset position, b, and her

[3]Unless stated otherwise, all variables are denoted in real terms.

[4]Note that our perfect substitutability result also holds if we assume different rules for the distribution of dividends. We will discuss this further in Section 3.

specific productivity level, z. The decision problem of a household h is given by:

$$V_t(b_{h,t}, z_{h,t}) = \max_{c_{h,t}, l_{h,t}, b_{h,t+1}} \left\{ \frac{c_{h,t}^{1-\gamma}}{1-\gamma} - \frac{l_{h,t}^{1+\psi}}{1+\psi} + \beta \sum_{z_{h,t+1}} Pr(z_{h,t+1}|z_{h,t}) V_{t+1}(b_{h,t+1}, z_{h,t+1}) \right\},$$

subject to equation (2). The Euler equation is given by:

$$c_{h,t}^{-\gamma} \geq \beta E_t \left\{ \left((1 + r_{t+1}) \frac{1 + \tau_t^C}{1 + \tau_{t+1}^C} \right) c_{h,t+1}^{-\gamma} \right\}, \tag{3}$$

which governs the intertemporal substitution decision of households. Both lower real interest rates and higher future consumption taxes increase the intertemporal policy wedge, $\frac{1}{1+r_{t+1}} \frac{1+\tau_{t+1}^C}{1+\tau_t^C}$, thereby incentivizing households to consume more today.

The labor-leisure equation is given by:

$$l_{h,t}^{\psi} = c_{h,t}^{-\gamma} z_{h,t} w_t \frac{1 - \tau_t^L}{1 + \tau_t^C}. \tag{4}$$

Consumption taxes and labor taxes directly influence the labor supply of households through the intratemporal policy wedge, $\frac{1-\tau_t^L}{1+\tau_t^C}$.

Let $c_t(b, z)$, $\ell_t(b, z)$, and $b_{t+1}(b, z)$ denote the policy functions for consumption, labor supply, and savings, respectively, that satisfy equations (2), (3), and (4) given the household's individual state.

2.2 Firms

Final good firms produce in a perfectly competitive market using intermediate goods as inputs. Their decision problem is:

$$\max_{y_{j,t}} \left\{ P_t Y_t - \int_0^1 p_{j,t} y_{j,t} dj \right\},$$

subject to a CES production technology:

$$Y_t = \left(\int_0^1 y_{j,t}^{1/\mu} dj \right)^{\mu},$$

where $y_{j,t}$ denotes the intermediate good produced by firm j and $p_{j,t}$ is the corresponding price. Y_t denotes the final consumption good, P_t denotes the overall price index, and μ determines the degree of substitution among input factors. The aggregate price index is given by:

$$P_t = \left(\int_0^1 p_{j,t}^{1/(1-\mu)} dj \right)^{1-\mu}.$$

Solving the maximization problem yields the demand function of final good firms for the intermediate good j:

$$y_{j,t} = \left(\frac{p_{j,t}}{P_t} \right)^{\frac{\mu}{1-\mu}} Y_t. \tag{5}$$

Intermediate goods are produced by a continuum of intermediate good firms in monopolistically competitive markets according to:

$$y_{j,t} = n_{j,t}.$$

Following Correia et al. (2013), we assume that price setting takes place before consumption taxes. As in Calvo (1983), we allow an intermediate good firm to reset its price only with a certain probability, θ. If a firm is allowed to reset its prices, it solves the following dynamic maximization problem:

$$\max_{p_t^*, \{y_{j,s}, n_{j,s}\}_{s=t}^{\infty}} \sum_{s=t}^{\infty} \beta^{s-t} (1-\theta)^{s-t} \left(\frac{p_t^*}{P_s} y_{j,s} - w_s n_{j,s} \right),$$

subject to the final good firms' demand given in (5). The optimal price ratio p_t^*/P_t that solves this problem is given by:

$$\frac{p_t^*}{P_t} = \frac{\sum_{s=t}^{\infty} \beta^{s-t} (1-\theta)^{s-t} \left(\frac{P_t}{P_s} \right)^{\frac{\mu}{1-\mu}} Y_s \mu w_s}{\sum_{s=t}^{\infty} \beta^{s-t} (1-\theta)^{s-t} \left(\frac{P_t}{P_s} \right)^{\frac{\mu}{1-\mu}} Y_s}. \tag{6}$$

For future reference, let $(1 + \pi_{t+1}) = \frac{P_{t+1}}{P_t}$ denote the gross inflation rate.

2.3 Policy

We close the model by specifying monetary and fiscal policy.

Monetary policy. To simplify our analytical results in Section 3, we assume for now that monetary policy directly controls the real interest rate, r_t. Importantly, our perfect substitutability result does not depend on this simplification and still holds if we assume that monetary policy controls the nominal interest rate. Note that given our timing notation, monetary policy sets r_{t+1} in period t such that r_t is predetermined in period t.

Fiscal policy. The government has expenditures for a fixed amount of government consumption, $\bar{G}$, lump-sum transfers, Tr_t, and for repaying debt, B_t. It finances its expenditures by collecting total tax payments, T_t, and by issuing future debt. The government's budget constraint is given by:

$$\bar{G} + Tr_t + (1 + r_t)B_t = B_{t+1} + T_t. \tag{7}$$

Total tax payments are given by:

$$T_t = \tau_t^C C_t + \tau_t^L w_t L_t, \tag{8}$$

where C_t and L_t denote aggregate consumption and aggregate labor, respectively. For simplicity, we assume for now $\bar{G} = 0$ but relax this assumption in Section 4.

2.4 Aggregation, Market Clearing, and Equilibrium

The aggregate production function of the economy is given by:

$$S_t Y_t = \int_0^1 n_{j,t} dj \equiv N_t, \tag{9}$$

where N_t denotes the aggregate labor demand of the intermediate good firms. S_t measures the efficiency loss that occurs whenever prices differ and is given by:

$$S_t \equiv \int_0^1 \left(\frac{p_{j,t}}{P_t} \right)^{\frac{\mu}{1-\mu}} dj \geq 1.$$

It evolves according to:

$$S_{t+1} = (1 - \theta) S_t (1 + \pi_{t+1})^{\frac{-\mu}{1-\mu}} + \theta \left(\frac{p^*_{t+1}}{P_{t+1}} \right)^{\frac{\mu}{1-\mu}}. \tag{10}$$

Inflation is a function of the optimal relative price of the updating firms:

$$1 + \pi_t = \left(\frac{1 - \theta}{1 - \theta (\frac{p^*_t}{P_t})^{\frac{1}{1-\mu}}} \right)^{1-\mu}. \tag{11}$$

The distribution of households over their individual states, $\Gamma_{t+1}(\mathcal{B}, z')$, evolves following the exogenous Markov chain for the productivity level and the endogenously derived savings policy functions of the households. Formally:

$$\Gamma_{t+1}(\mathcal{B}, z') = \int_{\{(b,z):b_{t+1}(b,z)\in\mathcal{B}\}} Pr(z'|z)d\Gamma_t(b,z) \tag{12}$$

for all sets $\mathcal{B} \subset \mathbb{R}$. Aggregate labor supply, consumption, and savings are:

$$L_t = \int_0^1 z\ell_t(b,z)d\Gamma_t(b,z), \tag{13}$$

$$C_t = \int_0^1 c_t(b,z)d\Gamma_t(b,z), \tag{14}$$

and

$$B^d_{t+1} = \int_0^1 b_{t+1}(b,z)d\Gamma_t(b,z), \tag{15}$$

respectively.

Labor market clearing requires:

$$L_t = N_t, \tag{16}$$

the bond market clears when:

$$B_t = B^d_t, \tag{17}$$

and the goods market clears when:

$$Y_t = C_t + \bar{G}. \tag{18}$$

Dividend payments are given by:

$$D_t = Y_t - w_t N_t. \tag{19}$$

Equilibrium. We define an equilibrium of the economy to consist of:

1. Policy and value functions $\{b_{t+1}(b,z),\ \ell_t(b,z),\ c_t(b,z),\ V_t(b,z)\}_{t=0}^{\infty}$ that solve the households' problems,

2. distributions $\{\Gamma_t(b,z)\}_{t=0}^{\infty}$ that evolve according to (12),

3. sequences of the aggregate variables

$$X \equiv \left\{C_t, L_t, N_t, Y_t, d_t, i_t, w_t, \pi_t, r_t, p_t^*/P_t, S_t, Tr_t, T_t, \tau_t^C, \tau_t^L, B_t^d, B_t\right\}_{t=0}^{\infty}$$

that satisfy the equilibrium conditions (6), (7), (8), (9), (10), (11), (16), (18), (19), the household aggregation equations (13), (14), (15), as well as the paths for the real interest rate, consumption taxes, labor taxes, and the government debt level to be specified below.

3 HANK-UFP

In this section, we prove that HANK-UFP is a perfect substitute for monetary policy in HANK. In particular, we derive a set of sufficient conditions for three *aggregate* fiscal instruments which jointly replicate the consumption and labor supply of *each* household and, thus, the allocation associated with any given change in interest rates.

3.1 Perfect Substitutability with Monetary Policy

Monetary policy. Assume a standard perfect foresight monetary policy experiment. The economy is in steady state when in period $t = 0$, monetary policy announces a new path of real rates, $\{r_t^{MP}\}_{t=1}^{\infty}$, with $r_t^{MP} = \bar{r}$ for all $t > s$ for some s. We denote variables associated with this monetary policy experiment with a superscript MP. Consumption taxes, labor taxes, and the government debt level are fixed at their steady state values, that is, for all t, $\tau_t^{L,MP} = \bar{\tau}^L$, $\tau_t^{C,MP} = \bar{\tau}^C$, $B_t^{MP} = \bar{B}$, while transfers, Tr_t^{MP}, adjust to keep the government budget balanced. We focus on the equilibrium in which the economy converges back to steady state for $t \to \infty$.

Monetary policy affects the economy through changing the optimization problem of households. In particular, it changes the households' problem in two ways: first, monetary policy changes the *intertemporal policy wedge*, $\frac{1}{1+r_{t+1}}\frac{1+\tau_{t+1}^C}{1+\tau_t^C}$, in the Euler equation of households (equation (3)) which incentivizes households to intertemporally reallocate consumption. Second, monetary policy has effects on the budget constraints of households. Importantly, these effects are not the same across households since, on the one hand, households differ in the composition of their income and, on the other hand, monetary policy has different effects on the various income components of households. We come back to this in the next paragraph.

HANK-UFP. We now describe how fiscal policy replicates the allocation associated with the monetary policy experiment. Assume that the real interest rate is kept at its steady state level, $r_t^{UFP} = \bar{r}\ \forall t$, and fiscal policy changes the paths for its aggregate instruments, $\tau_t^{C,UFP}$, $\tau_t^{L,UFP}$, and B_{t+1}^{UFP}. The following proposition states our perfect substitutability result between fiscal policy and monetary policy in HANK.

Proposition 1. *Consider HANK-UFP, a fiscal policy scheme which sets the paths for consumption taxes, $\tau_t^{C,UFP}$, labor taxes, $\tau_t^{L,UFP}$, and the government debt level, B_{t+1}^{UFP}, according to the following conditions*

$$\left(1+\bar{r}\right)\frac{1+\tau_t^{C,UFP}}{1+\tau_{t+1}^{C,UFP}} = 1 + r_{t+1}^{MP}, \textit{with } \tau_0^{C,UFP} = \bar{\tau}^C, \tag{20}$$

$$\frac{1-\tau_t^{L,UFP}}{1+\tau_t^{C,UFP}} = \frac{1-\bar{\tau}^L}{1+\bar{\tau}^C}, \tag{21}$$

$$B_{t+1}^{UFP} = \frac{1+\tau_t^{C,UFP}}{1+\bar{\tau}^C} B_{t+1}^{MP} \tag{22}$$

while lump-sum transfers, Tr_t^{UFP}, adjust to keep the government budget constraint balanced. HANK-UFP yields the same allocation as the monetary policy experiment. That is, consumption and labor supply are the same for each household in every period, i.e., $(c_{h,t}^{UFP}, l_{h,t}^{UFP}) = (c_{h,t}^{MP}, l_{h,t}^{MP})\ \forall h$ and $\forall t$. Hence, conditions (20) - (22) are sufficient conditions for HANK-UFP to be a perfect substitute for monetary policy.

While we relegate the formal proof of Proposition 1 to Appendix A, we now explain the rationale behind our perfect substitutability result. As in Correia et al. (2008) and Correia et al. (2013), HANK-UFP uses consumption taxes and labor taxes to replicate the effects of monetary policy on the policy wedges in the first-order conditions of households. According to condition (20), consumption taxes are set such that the intertemporal policy wedge in the Euler equation of each household is the same as in the monetary policy experiment. Intuitively, by changing the ratio of future over current consumption taxes, fiscal policy changes the relative price of current consumption versus future consumption. This way, fiscal policy triggers the same incentive to intertemporally reallocate consumption as a change in the real interest rate. While there are two possibilities to achieve equivalence in the intertemporal policy wedge with monetary policy, namely a pre-announced change in future consumption taxes *or* surprise changes in today's consumption taxes, the second part of condition (20) rules out surprise changes as these are not consistent with equivalence in the budget constraint as we will explain below.

Unlike a change in the real interest rate, adjusting consumption taxes changes the *intratemporal policy wedge*, $\frac{1-\tau_t^L}{1+\tau_t^C}$, in the labor-leisure equations of households (equation (4)). When labor taxes are set according to condition (21), they offset this effect on the labor supply of households.

Furthermore, HANK-UFP ensures that the effects on the budget constraint of each household are the same as with monetary policy. To this end, HANK-UFP replicates the effects of monetary policy on *each component* of households' incomes. Equivalence in the intratemporal policy wedge (condition (21)) ensures that households' net wage is the same in consumption value terms as in the monetary policy experiment. Condition (20) ensures that the real return on assets in consumption value terms is the same as in the monetary policy experiment. In particular, a pre-announced change in future consumption taxes changes $(1+\bar{r})\frac{b_{h,t+1}^{UFP}}{1+\tau_{t+1}^{C,UFP}}$ but it leaves $(1+\bar{r})\frac{b_{h,t}^{UFP}}{1+\tau_t^{C,UFP}}$ unchanged just like an interest rate change changes the consumption value of households' assets in the next period, $(1+r_{t+1}^{MP})\frac{b_{h,t+1}^{MP}}{1+\bar{\tau}^C}$, by changing its return, $1+r_{t+1}^{MP}$, but leaves the consumption value of households' assets in this period, $(1+r_t^{MP})\frac{b_{h,t}^{MP}}{1+\bar{\tau}^C}$, unchanged. As mentioned above, this is not feasible with a surprise change in consumption taxes today as this would already affect $(1+\bar{r})\frac{b_{h,t}^{UFP}}{1+\tau_t^{C,UFP}}$.

Equivalence in the real return on assets implies that households want to save the same amount in consumption value terms as in the monetary policy experiment, that is $b_{h,t+1}^{UFP} = \frac{1+\tau_t^{C,UFP}}{1+\bar{\tau}^C} b_{h,t+1}^{MP}$. Such savings positions also ensure that each household's asset income is the same as with monetary policy. However, to render it feasible for each household to increase her asset position by the factor $\frac{1+\tau_t^{C,UFP}}{1+\bar{\tau}^C}$, aggregate asset supply also needs to change by this factor which is the case given that debt dynamics follow condition (22). At the same time, debt policies following condition (22) shift the same resources measured in consumption value terms to the government as monetary policy. Conditions (20)-(22) together with the government budget constraint (7) then yield the following transfer path:

$$Tr_t^{UFP} = \frac{1+\tau_t^{C,UFP}}{1+\bar{\tau}^C} Tr_t^{MP} + D_t\left(\frac{1+\tau_t^{C,UFP}}{1+\bar{\tau}^C} - 1\right). \tag{23}$$

Hence, transfers follow the path of transfers associated with the monetary policy experiment adjusted for the change in consumption value. In addition, transfers compensate for the change in consumption value of the dividend income.[5] This reflects that tax revenues are different with HANK-UFP and monetary policy as the change in consumption taxes applies

[5]Note that the same compensation for the loss in consumption value of dividends can be achieved if we allow for a lump-sum transfer to firms. In this case, the aggregate transfer payment to firms would equal the second term in equation (23). Hence, our perfect substitutability result does not depend on the assumption of lump-sum dividends but can also be achieved with any other assumption on how dividends are distributed.

to total output, Y_t, whereas the change in labor taxes only applies to total labor income, $w_t N_t = Y_t - D_t$. Overall, equation (23) implies that also the lump-sum income component of households is the same in consumption value terms as with monetary policy.

In sum, HANK-UFP replicates the effects of monetary policy both on the policy wedges in the first-order conditions and on the budget constraints of households. Thus, each household faces the same optimization problem with HANK-UFP and monetary policy which implies that both policies induce the same consumption and labor supply of each household. As neither the interest rate nor fiscal policy variables are part of the firms' equilibrium equations, equivalence in each household's behavior generates the same allocation through general equilibrium.

Policy-induced redistribution. A corollary of our perfect-substitutability result is that monetary policy and HANK-UFP induce the same redistribution among households. We now show that HANK-UFP achieves this by replicating the redistribution of monetary policy through the policy block, that is, the partial equilibrium redistribution through changes in monetary and fiscal policy variables. To capture this policy-induced redistribution formally, we define the policy-exposure of each household by $\Xi_{h,t}$. This captures the partial-equilibrium changes of resources for household h which are induced by changes in policy variables assuming fixed households' and firms' behavior, that is, $(c_{h,t}, l_{h,t}, \frac{b_{h,t+1}}{1+\tau_t^C}, d_t, w_t) = (\bar{c}_h, \bar{l}_h, \frac{\bar{b}'_h}{1+\bar{\tau}^C}, \bar{d}, \bar{w})$.[6]

Lemma 1. *The policy-exposure of household h towards monetary policy is given by $\Xi_{h,t}^{MP} = \bar{B}(\bar{r} - r_t^{MP}) - \bar{b}_h(\bar{r} - r_t^{MP})$. Consider periods of expansionary monetary policy, that is $r_t^{MP} < \bar{r}$. Then $\Xi_{h,t}^{MP} < 0$ for $\bar{b}_h > \bar{B}$ and $\Xi_{h,t}^{MP} > 0$ for $\bar{b}_h < \bar{B}$. That is, expansionary monetary policy redistributes from households that have a higher asset position than the average to households that have a lower asset position than the average. The opposite is true for periods with contractionary monetary policy. With HANK-UFP, the households' policy exposure is given by $\Xi_{h,t}^{UFP} = \frac{1+\tau_t^{C,UFP}}{1+\bar{\tau}^C} \Xi_{h,t}^{MP}$. Hence, HANK-UFP replicates the policy exposure of monetary policy for each household in consumption value terms.*

Again, we relegate the proof of Lemma 1 to Appendix B and focus here on the intuition. A decrease in interest rates generates a negative wealth effect on assets from which households suffer in proportion to their asset holdings. At the same time, lower interest rates shift resources to the government as the government issues the assets and now has lower interest payments. Hence, lower interest rates imply a redistribution from asset holders to the

[6] The bar on top of aggregate variables denotes their respective steady state values. The bar on top of choice variables of households denotes household h's behavior in stationary equilibrium. As the savings of a household determine its individual state in the next period, we need to adjust the savings for the consumption value. Otherwise, this partial equilibrium decomposition would compare different households in the monetary policy experiment and in the HANK-UFP case as discussed above.

government. As this implies an increase in lump-sum transfers, expansionary monetary policy redistributes from asset-rich to asset-poor households.[7] HANK-UFP induces the same negative wealth effect on assets from which households suffer in proportion to their asset holdings—exactly as with monetary policy. The reason is that higher consumption taxes decrease the consumption value of assets thereby inflating away the buffer stock of households. As a consequence, households buy additional government debt in proportion to their asset holdings. Since this expansion in debt increases lump-sum transfers to all households, the redistribution through changes in policy variables is the same as with monetary policy.

Relation to perfect substitutability in RANK. There are three differences to the perfect substitutability result in RANK by Correia et al. (2013). First, consumption taxes and labor taxes set according to conditions (20) and (21), respectively, are sufficient for perfect substitutability with monetary policy in RANK. Our analysis shows that these two instruments are no longer sufficient in HANK. With only tax policies, fiscal policy replicates the effects of monetary policy on the policy wedges in the first-order conditions of households but not on their budget constraints such that households are differently exposed to these tax policies and monetary policy. In HANK, these heterogeneous effects on households' budget constraints matter. This is because first, the heterogeneous exposure of households prevents cross-sectional equivalence with monetary policy. Second, it also breaks the aggregate equivalence since households are heterogeneous in their MPCs. In addition, with only tax policies, the value of total assets in consumption value terms changes which has real effects since households hold these assets for self-insurance purposes. In our numerical analyses in Section 4.4, we quantify the shortcomings of this fiscal policy scheme and show them to be substantial.

Second, in RANK, it does not matter whether equivalence in the intratemporal policy wedge is achieved via a surprise change in today's consumption taxes or a pre-announced change in future consumption taxes as both alternatives are consistent with perfect substitutability with monetary policy. As explained above, these two alternatives have different dynamic effects on the asset income of households, but with a representative, permanent-income consumer these differences do not affect her behavior as long as the effects on the policy wedges in the first-order conditions are the same. In contrast, in HANK, only pre-announced changes of future consumption taxes are consistent with perfect substitutability as only these are consistent with equivalence in the sequence of budget constraints of each household.

[7] In Section 3.2, we show that our perfect substitutability result does not depend on the assumption that transfers adjust after a monetary policy shock but also holds when we assume any other fiscal response.

Third, in RANK, unconstrained monetary policy implements the first-best allocation if lump-sum transfers are available and, consequently, UFP replicates this first-best allocation. Unlike in RANK, unconstrained monetary policy does not necessarily implement the first-best allocation in our HANK framework and, thus, neither does HANK-UFP. Yet, Proposition 1 shows that fiscal policy can always at least implement the welfare associated with unconstrained monetary policy.

3.2 Model Extensions

We now first show that our perfect substitutability result does not depend on our assumption on the fiscal response to monetary policy and that second, it also holds when we extend our model by sticky wages and investment.

3.2.1 Alternative fiscal responses to monetary policy

The HANK literature highlights the importance of the fiscal response to monetary policy (see Kaplan et al. (2018) and Auclert et al. (2020)). In particular, Kaplan et al. (2018) distinguish three different fiscal responses: first, transfers adjust to balance the government budget after a change in monetary policy which is our baseline. Second, debt adjusts in the short-run and transfers ensure that government debt returns to its original steady state level in the long-run. Third, government spending adjusts. Our perfect substitutability result between fiscal policy and monetary policy does not depend on the fiscal response we assume. If debt adjusts in the short-run and transfers bring back government debt in the long-run in the monetary policy experiment, the conditions in Proposition 1 are still sufficient. If fiscal policy adjusts government spending in response to monetary policy, Proposition 1 needs to be extended by the following condition for government spending: $G_t^{UFP} = G_t^{MP}$.

3.2.2 Sticky Wages

We now show that Proposition 1 is still sufficient for perfect substitutability between HANK-UFP and monetary policy in a model in which both prices and wages are sticky. We model sticky wages as in Auclert et al. (2018). We here only sketch the modifications of the model and leave the details for Appendix C. Instead of each household deciding on her labor supply, labor hours are now determined by the labor demand of a continuum of monopolistically competitive unions. We assume quadratic utility costs of adjusting the nominal wage w_{kt} set by union k, by allowing for an extra additive disutility term $\frac{\nu}{2}\int_k \left(\frac{w_{kt}}{w_{kt-1}} - 1\right)^2 dk$ in households' utility (1). In every period t, union k sets a common wage for each of its members, and calls upon its members to supply hours according to a uniform rule, so that $l_{hkt} = L_{kt}$. The union sets w_{kt} to maximize the average utility of its members given this allocation rule. Real wages evolve according to $\frac{1+\pi_t^W}{1+\pi_t} = \frac{w_t}{w_{t-1}}$, where π_t^W is the nominal wage inflation which evolves according to an aggregate non-linear wage Phillips curve

$$\pi_t^w(1+\pi_t^w) = \frac{\epsilon_w}{\nu} \int L_t \left(L_t^{\psi} - \frac{\epsilon_w - 1}{\epsilon_w} \frac{1-\tau_t^L}{1+\tau_t^C} w_t z_{ht} c_{ht}^{-\gamma} \right) dh + \beta \pi_{t+1}^w (1+\pi_{t+1}^w), \tag{24}$$

where ϵ_w denotes the elasticity of substitution among unions.

Note that labor supply is now exogenous to households and, thus, every solution to the household problem now only needs to satisfy equations (2) and (3). Introducing sticky wages does not change either of the two equations compared to our baseline model with flexible wages. Labor supply is now pinned down by the wage Phillips curve (24) instead of households' individual labor-leisure equations. However, this Phillips curve is equally affected by HANK-UFP and by monetary policy given that condition (21) generates the same intratemporal policy wedge with both policies and given that each household consumes the same and, thus, has the same marginal utility of consumption with both policies. Hence, also with sticky wages, all equilibrium conditions are equally affected by HANK-UFP and by monetary policy and, thus, our perfect substitutability result holds.

3.2.3 Extension to investment

In this section, we show that fiscal policy can also be a perfect substitute for monetary policy if we extend our model by capital. To this end, we assume that intermediate goods firms produce according to a typical Cobb-Douglas production function with labor and capital as inputs (see Appendix D for details). We allow for a linear capital subsidy, τ_t^F, which is directly paid to the intermediate goods firms. The firms' first-order conditions yield the following term for real marginal cost:

$$mc_t = \left(\frac{1}{\alpha}\right)^{\alpha} \left(\frac{1}{1-\alpha}\right)^{1-\alpha} (r_t^{k,F} - \tau_t^F)^{\alpha} (w_t)^{1-\alpha}, \tag{25}$$

where α is the share of capital and $r_t^{k,F}$ is the rental rate of capital. An increase in capital subsidies increases the demand for capital in the same way as expansionary monetary policy since both policies equally affect the policy wedge in the firms' first-order conditions. Thus, HANK-UFP replicates the incentives to use capital induced by monetary policy by setting the capital subsidy according to the following condition (assuming $\tau_t^{F,MP} = \bar{\tau}^F = 0$ without loss of generality):

$$r_t^{k,F,MP} = r_t^{k,F,UFP} - \tau_t^{F,UFP}. \tag{26}$$

The economy's capital stock evolves according to $K_{t+1} = I_t + (1-\delta)K_t$, where δ is the depreciation rate of capital and the investment good, I_t, is produced with the same technology as the final consumption good. We assume that the depreciation of capital is replaced through maintenance and, thus, $r_t^K = r_t^{K,F} - \delta$ is the net return on capital. The capital stock is held by the households and, thus, monetary policy also affects households' budget constraints through changing the return on capital. The effects on households' budget constraints and, thus, the specification of HANK-UFP depend on the degree of substitutability between bonds and capital from the perspective of households. We next study a model in which capital is a perfect substitute to bonds and one in which capital is illiquid and, thus, an imperfect substitute to bonds.

Bonds and capital as perfect substitutes. We start by specifying the conditions for HANK-UFP when capital is a perfect substitute for bonds. Perfect substitutability of both assets implies that the return on both assets is the same and, hence, $r_t = r_t^k$. In this case, the budget constraint of household h is given by:

$$c_{h,t} = (1+r_t)\frac{b_{h,t}+k_{h,t}}{1+\tau_t^C} - \frac{b_{h,t+1}+k_{h,t+1}}{1+\tau_t^C} + \frac{1-\tau_t^L}{1+\tau_t^C} w_t z_{h,t} l_{h,t} + \frac{D_t + Tr_t}{1+\tau_t^C} \tag{27}$$
$$b_{h,t+1}, k_{h,t+1} \geq 0,$$

where $k_{h,t}$ is the amount of capital held by household h.[8]

Consider the following modified condition for the government debt level:

$$B_{t+1}^{UFP} = \frac{1+\tau_t^{C,UFP}}{1+\bar{\tau}^C}\bar{B} + \left(\frac{1+\tau_t^{C,UFP}}{1+\bar{\tau}^C} - 1\right) K_{t+1}^{MP}. \tag{28}$$

HANK-UFP consisting of conditions (20), (21), (28), and (26) is a perfect substitute for monetary policy. We relegate the proof to Appendix D.5 and focus here on the intuition of this result.

Equivalence in the intertemporal policy wedge in the Euler equations of households im-

[8]Given $r_t^k = r_t$, the portfolio of each household is indeterminate. For simplicity, we assume that all households have the same bond to capital ratio of $\bar{B}/\bar{K}$ but our results do not depend on this assumption.

plies that each household wants to save the same amount in consumption value terms:

$$b_{h,t+1}^{UFP} + k_{h,t+1}^{UFP} = \left(\frac{1+\tau_t^{C,UFP}}{1+\bar{\tau}^C}\right)(b_{h,t+1}^{MP} + k_{h,t+1}^{MP}). \tag{29}$$

When the savings of each household evolve according to (29), the budget constraint of each household is the same with HANK-UFP and with monetary policy. Setting the government debt level according to condition (28) implies that $K_t^{UFP} = K_t^{MP}$ and that equation (29) is feasible for each household.

Hence, in addition to replicating the incentives for firms to use capital, HANK-UFP now also replicates the redistribution from capital holders to firms induced by monetary policy: households compensate for the loss in consumption value of their capital holdings by buying more bonds in proportion to their capital holdings. These additional bonds, in turn, finance the capital subsidies to firms.

Bonds and capital as imperfect substitutes. We now assume that capital holdings are subject to an illiquidity friction which renders capital an imperfect substitute for bonds (as for example in Kaplan et al. (2018) and Bayer et al. (2019)). In this case, $r_t^k = r_t + \epsilon_t^k$, where ϵ_t^k is an endogenous and time-varying spread on illiquid capital that households demand because capital is less liquid and, thus, less suited for self-insurance purposes. As Bayer et al. (2023) show, ϵ_t^k also depends on the amount of bonds. As a consequence, HANK-UFP cannot compensate for the loss in consumption value of savings in capital by increasing the bond supply but it needs to use an additional instrument. More specifically, HANK-UFP needs to issue an additional asset which we label *synthetic capital*, Ω, which has the same pay-off structure and the same illiquidity friction as capital. In other words, synthetic capital is a perfect substitute for capital from the households' perspective but it is not used in production. Accordingly, the government budget constraint is given by:

$$Tr_t + \tau_t^F K_t + (1+r_t^k)\Omega_t + (1+r_t)B_t = B_{t+1} + \Omega_{t+1} + T_t, \tag{30}$$

and without loss of generality, we assume that $\Omega_t^{MP} = \bar{\Omega} = 0$.

The budget constraint of household h is then given by:

$$c_{h,t} = \frac{1+r_t}{1+\tau_t^C} b_{h,t} - \frac{b_{h,t+1}}{1+\tau_t^C} + \frac{1+r_t^K}{1+\tau_t^C}(k_{h,t}+\omega_{h,t}) - \frac{(k_{h,t+1}+\omega_{h,t+1})}{1+\tau_t^C} + \frac{1-\tau_t^L}{1+\tau_t^C} w_t z_{h,t} l_{h,t} + \frac{D_t + Tr_t}{1+\tau_t^C}, \qquad b_{h,t+1}, k_{h,t+1} \geq 0, \tag{31}$$

where $\omega_{h,t+1}$ is the amount of synthetic capital household h holds.

Consider the following condition for the issuance of synthetic capital:

$$\Omega_{t+1}^{UFP} = \left(\frac{1+\tau_t^{C,UFP}}{1+\bar{\tau}^C} - 1 \right) K_{t+1}^{MP}. \tag{32}$$

HANK-UFP consisting of conditions (20), (21), (22), (26), and (32) is a perfect substitute for monetary policy. We relegate the formal proof to Appendix D.6 and focus here on the intuition.

Equivalence with monetary policy in the real return on illiquid assets implies that each household wants to save the same amount of illiquid assets in consumption value terms:

$$\omega_{h,t+1}^{UFP} + k_{h,t+1}^{UFP} = \left(\frac{1+\tau_t^{C,UFP}}{1+\bar{\tau}^C} \right) (\omega_{h,t+1}^{MP} + k_{h,t+1}^{MP}). \tag{33}$$

Given that households' portfolios evolve according to $\left(b_{h,t+1}^{UFP}, k_{h,t+1}^{UFP} + \omega_{h,t+1}^{UFP}\right) = \left(\frac{1+\tau_t^{C,UFP}}{1+\bar{\tau}^C} b_{h,t+1}^{MP}, \frac{1+\tau_t^{C,UFP}}{1+\bar{\tau}^C}(k_{h,t+1}^{MP} + \omega_{h,t+1}^{MP})\right)$, the budget constraint of each household is equally affected by HANK-UFP and monetary policy. Setting synthetic capital according to condition (32) implies that $K_t^{UFP} = K_t^{MP}$ and that equation (33) is feasible for each household. This way, HANK-UFP also replicates the redistribution from capital holders to firms: households hold the synthetic capital in proportion to their capital holdings, which, in turn, finances the capital subsidy.

4 Circumventing the ELB Constraint

Our perfect substitutability result between fiscal policy and monetary policy is especially relevant when monetary policy is constrained. To illustrate this, we now show how HANK-UFP circumvents the ELB constraint—a natural example of constrained monetary policy. In this section, we first demonstrate that HANK-UFP achieves the same allocation as hypothetically unconstrained monetary policy when a discount factor shock pushes the economy to the ELB. To this end, we now study a numerical exercise of our model in Section 2 including sticky wages as described in Appendix C. We then show that if the UFP measure does not include debt policies but only consists of tax policies, output and inflation fall by more than with unconstrained monetary policy while the ELB binds and the economy converges to a new steady state after the ELB stops binding.

Monetary policy. We now assume that monetary policy controls the nominal interest rate and follows a Taylor rule. The central bank sets the nominal interest rate between period t and $t+1$, i_{t+1}, according to:

$$1 + i_{t+1} = \max \left\{ \underline{I}, (1+\bar{i}) \left(\frac{\pi_t}{\bar{\pi}} \right)^{\phi_\pi} \left(\frac{Y_t}{\bar{Y}} \right)^{\phi_Y} \right\}. \tag{34}$$

The parameters ϕ_π and ϕ_Y measure how responsive the central bank reacts to deviations in inflation and output, respectively, from steady state. In the case of constrained monetary policy, the Taylor rule is truncated by the ELB. Thus, $\underline{I} = 1$ and nominal interest rates cannot go below zero. In the counterfactual of unconstrained monetary policy, $\underline{I} \longrightarrow -\infty$, and monetary policy follows the Taylor rule without any constraints.

The nominal and the real interest rate are linked via the Fisher equation:

$$1 + r_{t+1} = \frac{1 + i_{t+1}}{1 + \pi_{t+1}}. \tag{35}$$

4.1 Calibration

Table 1 summarizes our calibration which are standard values in the literature. We set the households' discount factor, β, such that the annual steady state real interest rate, $\bar{r}$, is 2%. We set both the coefficient for risk aversion, γ, and for the inverse Frisch elasticity, ψ, to 2. The latter reflects the finding of Chetty (2012). Following Christiano et al. (2011), we set the markup parameter, μ, to 1.2, and the price reversion rate, θ, to 0.15. For the degree of wage stickiness, we follow Auclert et al. (2018). In particular, we set the elasticity of substitution among union-specific tasks, ϵ^W, to 11 and the parameter governing the disutility of resetting wages, ν, to 100. If we linearized the wage Phillips curve, these values would imply a slope of 0.1 as in Auclert et al. (2018).

The calibration of the idiosyncratic income risk follows McKay et al. (2016). We assume that households cannot borrow. We choose the labor income risk to approximate the findings of Floden and Lindé (2001). We discretize a quarterly AR(1) process with an autoregres-

Table 1: Calibration of the model.

Parameter	Description	Value
β	Discount factor	0.982
γ	Risk aversion	2
ψ	Inverse of Frisch elasticity	2
μ	Markup	1.2
θ	Price reversion rate	0.15
ϵ^W	Elasticity of substitution among unions	11
ν	Disutility of resetting wage	100
ρ_z	Autocorrelation of idiosyncratic risk	0.966
σ_z	Unconditional variance of idiosyncratic risk	0.501
$\bar{\tau^C}$	Consumption tax rate	5%
$\bar{\tau^L}$	Labor tax rate	28%
$\bar{G}/\bar{Y}$	Government consumption share	0.2
$\bar{Tr}/\bar{Y}$	Transfer share	0.055
$\bar{B}/(4 * \bar{Y})$	Government debt share	0.9
ϕ_π	Inflation Taylor weight	1.5
ϕ_Y	Output Taylor weight	0.125

sive coefficient of 0.966 and an innovation variance of 0.017 into a Markov chain by using Rouwenhorst (1995)'s method.[9] The resulting Markov chain matches the unconditional and the conditional mean, the unconditional and the conditional variance, and the first-order autocorrelation of the underlying quarterly AR(1) process. For our discretization, we choose an 11-state Markov chain as for example in Auclert et al. (2018).

Following Correia et al. (2013), we set the consumption tax rate, $\bar{\tau}^C$, to 5% and the labor tax rate, $\bar{\tau}^L$, to 28%. As in Christiano et al. (2011), we set government consumption $\bar{G}/\bar{Y} = 0.2$. We set the government debt level to target a quarterly average MPC of 0.16 as in Kaplan et al. (2018). This results in an annual government debt share, $\bar{B}/(4 * \bar{Y})$, of 90%. Balancing the government budget then requires a steady state transfer share of $\bar{Tr}/\bar{Y} = 0.06$. We set the Taylor coefficient on inflation and output to 1.5 and 0.125, respectively, as it is standard in the literature.

4.2 Solution Method

We solve the model using the perfect foresight method proposed in McKay et al. (2016). We compute the transition paths of the economy in response to a discount factor shock. Initially, the economy is in steady state. Without fiscal policy interventions, we assume that the economy returns to its old steady state after 250 periods. With HANK-UFP, we assume

[9] Floden and Lindé (2001) estimate the annual log wage process assuming that it follows an AR(1) process resulting in an autoregressive coefficient of 0.961 and an innovation variance of 0.426. The annual AR(1) process is simulated by a quarterly AR(1) process with an autoregressive coefficient of 0.966 and an innovation variance of 0.017.

that the economy has transitioned to its new steady state after 250 periods.

We guess the paths of the prices and the quantities of the variables specified in Section 2.4. We then check whether these prices and quantities are consistent with the definition of an equilibrium in Section 2.4 in each period. This implies to solve for the aggregate behavior of households given the guessed prices in each period. We use the endogenous grid point method of Carroll (2006) to solve the individual household problem backwards. We use the non-stochastic simulation algorithm in Young (2010) to simulate the distribution of households forward. When the aggregate behavior of households is not consistent with the guessed quantities, we update the guess for prices and quantities.[10]

4.3 HANK-UFP at the ELB

We follow Christiano et al. (2011) and approximate the effects of a binding ELB by engineering an unexpected temporary increase in the discount factor of households. The discount factor increases by 1.45% for 8 quarters before it jumps back to its steady state level. This brings the economy to the ELB which then binds for 5 quarters. The black, dash-dot lines in Figure 1 show the dynamics of macroeconomic aggregates in the *constrained monetary policy case*. In this case, we assume that monetary policy is constrained by the ELB and that there is no fiscal stimulus in the sense that taxes and the government debt level stay constant. Output falls by 3.9%, consumption by 4.9%, and inflation by 3.5 annual percentage points.

How would macroeconomic aggregates react if monetary policy was not constrained by the ELB? The red, solid lines show this *unconstrained monetary policy case*. Without the ELB constraint, the central bank sets negative interest rates for 5 quarters which decrease at most to -2.8 annual percentage points. Output now only falls by 2.2%, consumption by 2.8%, and inflation by 2.4 annual percentage points.

The blue, dashed lines show the *HANK-UFP case*. The impulse response functions (IRFs) of the macroeconomic aggregates reflect that HANK-UFP achieves the same stabilization as unconstrained monetary policy since both responses lie perfectly on top of each other. Fiscal policy sets the paths for consumption taxes, labor taxes, and the government debt level to replicate the effects of hypothetically unconstrained monetary policy on both the first-order conditions and on the budget constraint of each household. According to condition (20), consumption taxes increase while the ELB is binding along a pre-announced path, in total from 5.0% to 7.1%. This way, consumption taxes replicate the effects through the

[10]We use an auxiliary model to update our guesses. It approximates the aggregate behavior of households with an auxiliary Euler equation and an auxiliary aggregate wage Phillips curve which contain time-varying heterogeneity wedges. We solve the auxiliary model with a version of Newton's method and iterate until the aggregate behavior of households is consistent with the guessed quantities and prices.

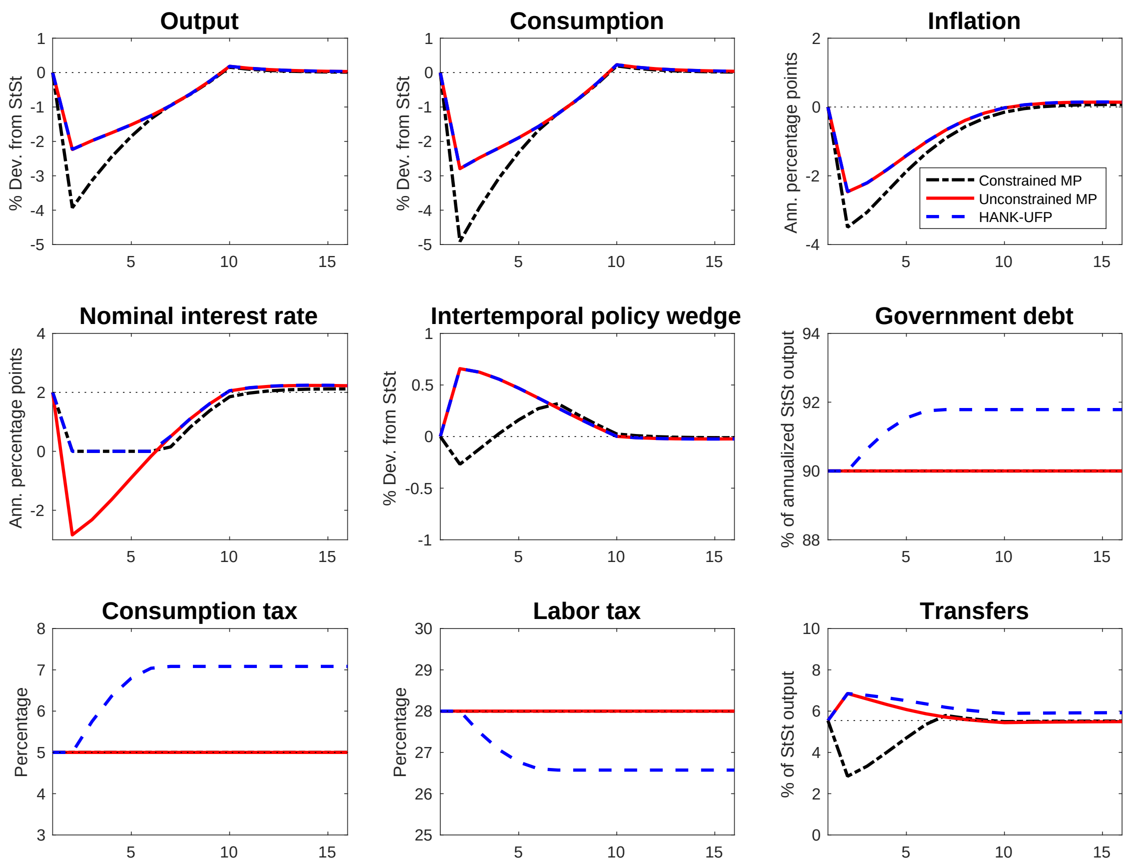

Figure 1: Impulse response functions after a shock to the discount factor with a Taylor rule truncated by the ELB ("Constrained MP"), with a Taylor rule without a lower bound ("Unconstrained MP"), and with a truncated Taylor rule and an additional HANK-UFP stimulus ("HANK-UFP"). Horizontal axes denote quarters.

intertemporal substitution channel of unconstrained monetary policy which is reflected by the IRFs of the intertemporal policy wedge in both cases. Labor taxes, correspondingly, decrease in total from 28.0% to 26.6% (condition (21)). In line with condition (22), government debt increases to a higher level of 91.8% (instead of 90.0%) such that households can hold the same amount of assets in consumption value terms as in the unconstrained monetary policy case. As equation (23) shows, this implies that transfers follow the path of transfers in the unconstrained monetary policy case but overshoot them to compensate for the loss in consumption value of the lump-sum income component. At most, transfers increase from 5.5% to 6.9% of GDP.

Cross-sectional equivalence. Section 3 shows that HANK-UFP replicates monetary policy by replicating the consumption and labor supply of each household in every period. Obviously, this result also holds in our numerical example. Hence, the welfare of each household and the paths for consumption inequality are the same. The equivalence also holds for the paths of wealth and income inequality when adjusting wealth and income for consumption value terms.

Return to steady state. Given repeated ELB crises, the labor tax might eventually become negative. To avoid that the labor tax turns into a labor subsidy, HANK-UFP can be reversed in good times once the constraint on interest rates is relaxed. This requires a decreasing path for consumption taxes, an increasing path for labor taxes, and a decrease in the government debt level accompanied by expansionary monetary policy. In other words, reversing HANK-UFP allows the fiscal authority to bring back the three fiscal instruments to their original steady state level.

4.4 Importance of Debt Policies

We now analyze the fiscal policy measure of Correia et al. (2013) —which only consists of tax policies—through the lens of our HANK model. Consumption taxes and labor taxes are set as in Proposition 1 to replicate the effects on the policy wedges in the first-order conditions of households while government debt is not adjusted for the consumption value but the same as in the unconstrained monetary policy case. Figure 2 compares this *RANK-UFP* measure (blue, dashed lines) with unconstrained monetary policy (red, solid lines) in response to the same discount factor shock as in Section 4.3. Note that the unconstrained monetary policy case as well as the constrained monetary policy case are the same as in Figure 1.

There are two take-aways: first, RANK-UFP does not achieve the same stabilization as unconstrained monetary policy while the ELB is binding as reflected in Figure 2. The

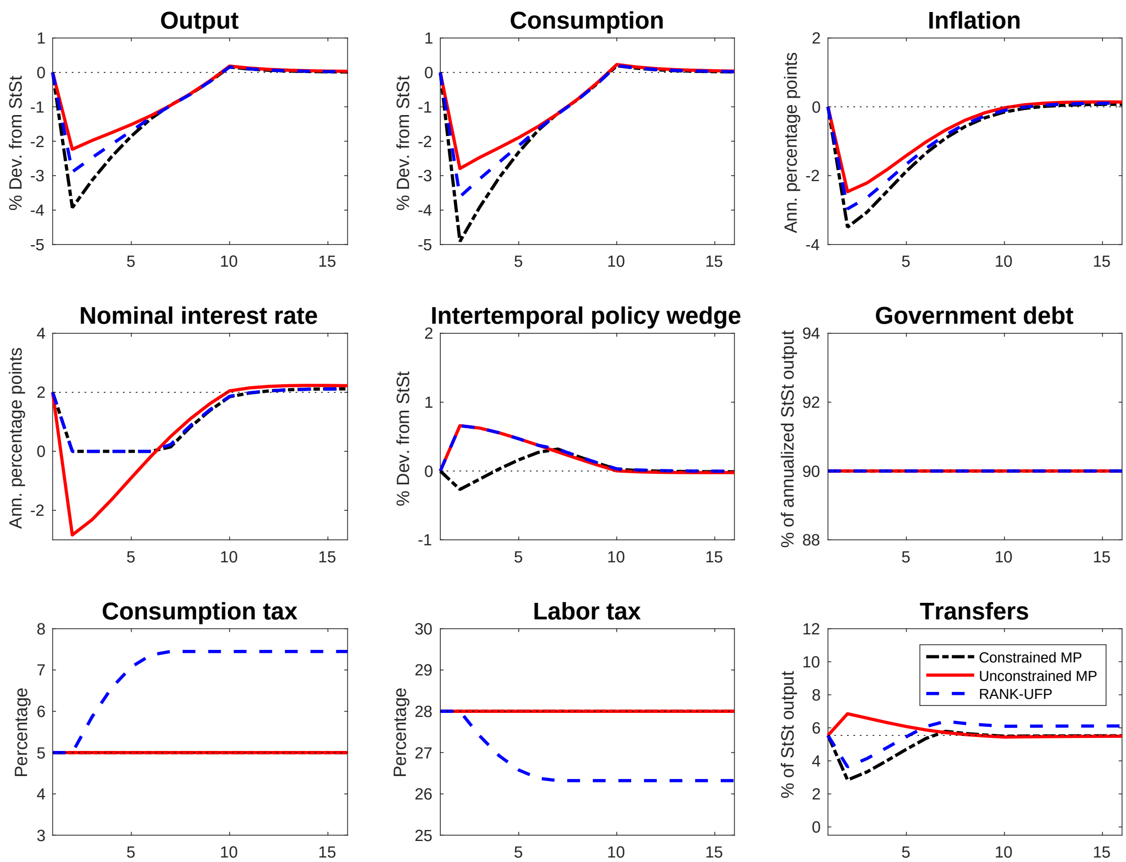

Figure 2: Impulse response functions after a shock to the discount factor with a Taylor rule without a lower bound ("Unconstrained MP") and with a truncated Taylor rule and an additional RANK-UFP stimulus ("RANK-UFP"). Horizontal axes denote quarters.

reason is that lump-sum transfers are lower than with HANK-UFP while the ELB binds since government debt does not increase in these periods. Hence, RANK-UFP does not provide additional resources to high-MPC households while the ELB is binding.[11] Accordingly, output drops on impact by -2.9%, consumption by -3.6%, and inflation by -3.0 annual percentage points.

The second take-away is that in the long-run, the real interest rate does not converge back to its original steady state level of 2% annually, but converges to a lower steady state level of 1.9% annually. The reason is that the supply of assets in consumption value terms is lower than in the unconstrained monetary policy case since government debt does not increase. Hence, households cannot hold the same amount of savings in consumption value terms and, hence, they are worse insured against their idiosyncratic income risk. This increases the inefficiencies from incomplete markets which corresponds to the negative effect of lower asset supply highlighted by Guerrieri and Lorenzoni (2017). As we will show below, this has a sizeable detrimental impact on households' welfare.[12] Our analysis shows that in HANK models, tax policies that interact with the precautionary savings motive of households are not neutral but can have a quantitatively significant impact on the economy in the long-run. Accordingly, debt policies play a crucial role in balancing these effects. This is in stark contrast to RANK models in which there is no precautionary savings motive and the asset demand is perfectly elastic with respect to the real interest rate in the long-run.[13]

Cross-sectional outcomes. As macroeconomic dynamics are not the same, cross-sectional outcomes also differ with RANK-UFP. We summarize the cross-sectional differences by comparing the welfare implications of RANK-UFP and HANK-UFP on each household. We compute the consumption compensation of each household in the economy with RANK-UFP and with HANK-UFP (which is the same as with unconstrained monetary policy).[14] Our welfare computation shows that each household is worse-off with RANK-UFP than with HANK-

[11] The stimulative effect of transfer policies is a common feature in HANK models as Ricardian equivalence does not hold (Oh and Reis, 2012, Hagedorn et al., 2019b, Bayer et al., 2020, Wolf, 2021).

[12] In a previous version of the paper, we assumed flexible wages in our numerical example. In that case, the worsened insurance possibilities of households due to RANK-UFP also result in a decrease in the effective labor supply of households and, thus, in a lower steady state output. With sticky wages, this effect is infinitesimal due to the assumption that labor supply is determined by unions.

[13] The convergence to a new steady state can be prevented if RANK-UFP is reversed after the ELB stops binding. This would imply a decreasing path of consumption taxes and an increasing path of labor taxes supported by expansionary monetary policy. Yet, given the non-equivalence of RANK-UFP and monetary policy, this reversal would again not be neutral on the allocation.

[14] We compute the consumption compensation as the consumption increase that is additionally necessary in the baseline of the constrained monetary policy case such that each household is indifferent between the baseline and the two policy cases (RANK-UFP and HANK-UFP). Given our specification of preferences, we cannot compute lifetime consumption compensation. Thus, we compute the consumption compensation for 4 quarters as in Kekre (2021).

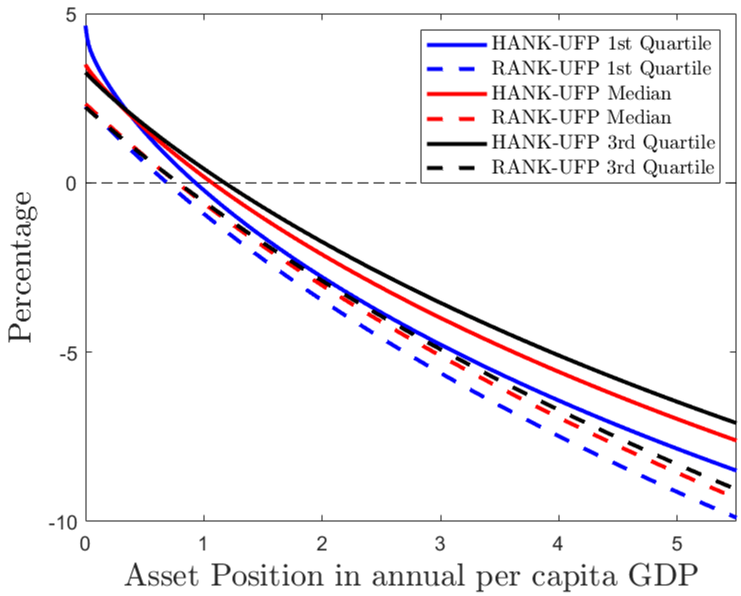

Figure 3: Consumption compensation for 4 quarters for each household such that she is indifferent between the respective policy and constrained monetary policy.

UFP, independent of her idiosyncratic state. Figure 3 shows this for a subset of households. It depicts the consumption compensation along the wealth distribution with RANK-UFP and with HANK-UFP for the households whose productivity levels correspond to the 1st quartile (blue), the median (red), and the 3rd quartile (black) of the productivity distribution. The solid lines depict the consumption compensation in the HANK-UFP case and the respective dashed lines depict the consumption compensation in the RANK-UFP case. Figure 3 shows that the solid lines always lie above the respective dashed lines indicating the welfare gain of each of these households with HANK-UFP compared to RANK-UFP.[15] Overall, our welfare analysis highlights that adding debt to the fiscal policy mix induces large welfare gains for each household.

[15]Note that all lines of both RANK-UFP and HANK-UFP cross the x-axis at some point. This indicates that high-asset households are worse off with the HANK-UFP stabilization (and, equivalently, with the unconstrained monetary policy stabilization). The reason is that the stabilization policy reduces their wealth significantly in consumption value terms. This effect outweighs their welfare loss out of a recession caused by the ELB in the baseline case of constrained monetary policy. Yet, the average consumption compensation of HANK-UFP (and equivalently unconstrained monetary policy) is 1.42% reflecting that HANK-UFP would be highly beneficial from a Utilitarian social planner perspective compared to RANK-UFP which only has an average consumption compensation of 0.02%.

5 Conclusion

We show that fiscal policy can be a perfect substitute for monetary policy in a HANK model as it can replicate the allocation of monetary policy. The insight is that by changing consumption taxes, labor taxes, and the government debt level, fiscal policy can manipulate the optimization problem of each household in the same way as a change in interest rates: these tax and debt policies jointly replicate the effects of interest rate changes on the policy wedges in the first-order conditions and the budget constraint of each household. Our perfect substitutability result is especially relevant when monetary policy is constrained—be it due to a binding lower bound, a currency union, or an exchange rate peg—since it implies that fiscal policy can circumvent these constraints. Unlike analyses with a representative agent, our analysis shows that including debt policies in the fiscal policy mix is necessary for perfect substitutability with monetary policy in HANK. Moreover, we highlight at the ELB that not using debt policies has quantitatively important consequences for cross-sectional and aggregate outcomes. When the fiscal authority uses only tax policies, it cannot replicate macroeconomic aggregates in the short-run and induces detrimental effects in the long-run.

We conclude by pointing out two avenues for future research. First, we have shown that a fiscal policy consisting of consumption taxes, labor taxes, and the government debt level can always implement the welfare of unconstrained monetary policy in HANK. Our conjecture is that fiscal policy can generate a higher welfare than unconstrained monetary policy. Second, we prove our perfect substitutability result in a perfect foresight environment. This means it applies to models without aggregate risk or equivalently to the model's first-order perturbation with aggregate risk (see Boppart et al. (2018) and Auclert et al. (2021)). Our conjecture is that our result still holds with higher-order aggregate risk. The reason is that higher-order aggregate risk might change the path of interest rates that the central bank wants to implement. However, the effects of this path of interest rates on the households' optimization problems would still be replicated by our fiscal policy scheme as our fiscal policy scheme replicates the allocation of any path of interest rates. We leave a formal analysis with higher-order aggregate risk for future research.

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A Proof of Proposition 1

In this section, we prove Proposition 1 which states that HANK-UFP yields the same allocation as the one induced by the real interest rate path in the monetary policy experiment. Let us assume that the equilibrium path induced by the monetary policy experiment in Section (3) is

$$X^{MP} = \left\{ B_t^{d*}, C_t^*, L_t^*, Y_t^*, D_t^*, w_t^*, \pi_t^*, \tilde{p}_t^*/P_t^*, r_t^{MP}, \bar{\tau}^C, \bar{\tau}^L, Tr_t^{MP}, \bar{B} \right\}_{t=0}^{\infty},$$

with the individual behavior of each household given by

$$x_h^{MP} = \{b_{h,t+1}^*, c_{h,t}^*, l_{h,t}^*\}_{t=0}^{\infty}.$$

We now show that

$$X^{UFP} = \left\{ \frac{1+\tau_{t-1}^{C,UFP}}{1+\bar{\tau}^C} B_t^{d*}, C_t^*, L_t^*, Y_t^*, D_t^*, w_t^*, \pi_t^*, \tilde{p}_t^*/P_t^*, \bar{r}, \tau_t^{C,UFP}, \tau_t^{L,UFP}, Tr_t^{UFP}, B_t^{UFP} \right\}_{t=0}^{\infty}$$
$$\text{with } x_h^{UFP} = \left\{ \frac{1+\tau_t^{C,UFP}}{1+\bar{\tau}^C} b_{h,t+1}^*, c_{h,t}^*, l_{h,t}^* \right\}_{t=0}^{\infty}$$

is an equilibrium path if $\tau_t^{L,UFP}, \tau_t^{C,UFP}$, and B_t^{UFP} satisfy conditions (20), (21), and (22), respectively.

Since neither the real interest rate nor the fiscal policy variables show up in any equilibrium condition of the firm side, it is sufficient to show that X^{UFP} satisfies the sequences of Euler equations (3), labor-leisure equations (4), and budget constraints (2) of each household as well as the government budget constraint. Without loss of generality, fix a household j. We now prove that if her behavior in the monetary policy experiment, $x_j^{MP} = \{b_{j,t+1}^*, c_{j,t}^*, l_{j,t}^*\}_{t=0}^{\infty}$, satisfies her sequences of Euler equations, labor leisure equations, and budget constraints in the monetary policy experiment, $x_j^{UFP} = \{\frac{1+\tau_t^{C,UFP}}{1+\bar{\tau}^C} b_{j,t+1}^*, c_{j,t}^*, l_{j,t}^*\}_{t=0}^{\infty}$ does so in the HANK-UFP case.

Satisfying household's first-order conditions. Any path of consumption, $\{c_{j,t}^*\}_{t=0}^{\infty}$, that satisfies the sequence of Euler equations of household j with $\{r_t^{MP}\}_{t=0}^{\infty}$ and steady state tax rates, also satisfies the sequence of Euler equations of household j with interest rates in steady state and $\{\tau_t^{C,UFP}\}_{t=0}^{\infty}$ which satisfies condition (20). In addition, any paths of consumption and labor, $\{c_{j,t}^*, l_{j,t}^*\}_{t=0}^{\infty}$, that satisfy the sequence of labor-leisure equations of household j with steady state taxes, satisfy the sequence of labor-leisure equations of household j if $\{\tau_t^{L,UFP}, \tau_t^{C,UFP}\}_{t=0}^{\infty}$ satisfy condition (21).

Satisfying household's budget constraint. Next, we show that if x_j^{MP} satisfies the sequence of budget constraints of household j in the monetary policy experiment, x_j^{UFP} does so with HANK-UFP. To this end, it is convenient to look at her three income components separately:

$$c_{j,t} = \underbrace{\frac{1-\tau_t^L}{1+\tau_t^C} w_t l_{j,t} z_{j,t}}_{I} + \underbrace{\frac{D_t + Tr_t}{1+\tau_t^C}}_{II} + \underbrace{(1+r_t)\frac{b_{j,t}}{1+\tau_t^C} - \frac{b_{j,t+1}}{1+\tau_t^C}}_{III}. \tag{36}$$

We now show that each of these components is exactly the same with x_j^{MP} and X^{MP} as well as with x_j^{UFP} and X^{UFP}.

The labor income, term (I) in equation (36), is the same in the monetary policy experiment and in the HANK-UFP case, iff:

$$\frac{1-\bar{\tau}^L}{1+\bar{\tau}^C} w_t^* l_{j,t}^* z_{j,t} = \frac{1-\tau_t^{L,UFP}}{1-\tau_t^{C,UFP}} w_t^* l_{j,t}^* z_{j,t} \tag{37}$$

which holds, given that taxes are set consistent with condition (21).

The lump-sum income, term (II) in equation (36), is identical in the monetary policy experiment and in the HANK-UFP case, iff:

$$\frac{D_t^* + Tr_t^{MP}}{1+\bar{\tau}^C} = \frac{D_t^* + Tr_t^{UFP}}{1+\tau_t^{C,UFP}} \tag{38}$$

which holds if Tr_t^{UFP} is set according to condition (23). We will show below that this is indeed the transfer path arising in the HANK-UFP case.

Finally, the asset income, term (III) in equation (36), is the same iff:

$$(1+r_t^{MP})\frac{b_{j,t}^*}{1+\bar{\tau}^C} - \frac{b_{j,t+1}^*}{1+\bar{\tau}^C} = (1+\bar{r})\frac{b_{j,t}^* \frac{1+\tau_{t-1}^{C,UFP}}{1+\bar{\tau}^C}}{1+\tau_t^{C,UFP}} - \frac{b_{j,t+1}^* \frac{1+\tau_t^{C,UFP}}{1+\bar{\tau}^C}}{1+\tau_t^{C,UFP}}. \tag{39}$$

Hence, we obtain equivalence iff:

$$(1+r_t^{MP})\frac{b_{j,t}^*}{1+\bar{\tau}^C} = (1+\bar{r})\frac{b_{j,t}^* \frac{1+\tau_{t-1}^{C,UFP}}{1+\bar{\tau}^C}}{1+\tau_t^{C,UFP}}$$

Using condition (20), we get

$$(1+r_t^{MP})\frac{b_{j,t}^*}{1+\bar{\tau}^C} = (1+\bar{r})\frac{b_{j,t}^*\frac{(1+r_t^{MP})}{1+\bar{\tau}^C}}{1+\bar{r}}$$
$$\iff b_{j,t}^* = b_{j,t}^*.$$

Thus, equation (45) holds. Hence, x_j^{UFP} satisfies the sequence of budget constraints of household j with HANK-UFP if x_j^{MP} does so in the monetary policy experiment. Furthermore, this also implies that if the individual state of household j is $(b_{j,t}^*, z_{j,t})$ in a given t in the monetary policy experiment, it is $(\frac{1+\tau_{t-1}^C}{1+\bar{\tau}^C}b_{j,t}^*, z_{j,t})$ with HANK-UFP.

In sum, $x_{h,t}^{UFP}$ and X^{UFP} are consistent with each household's problem if $x_{h,t}^{MP}$ and X^{MP} are. That is, each household consumes and works the same with HANK-UFP and in the monetary policy experiment and saves the same amount in consumption value terms.

Satisfying government's budget constraint. We now show that if X^{MP} satisfies the government's budget constraint, X^{UFP} does so as well. In the monetary policy experiment, the government's budget constraint is given by:

$$Tr_t^{MP} + r_t^{MP}\bar{B} = T_t^{MP}. \tag{40}$$

In the HANK-UFP case, the government's budget constraint is given by:

$$Tr_t^{UFP} + (1+\bar{r})B_t^{UFP} = B_{t+1}^{UFP} + T_t^{UFP}.$$

We show that given Proposition 1, this is exactly the same as equation (40). Assuming the transfer path given by condition (23), plugging in condition (22), using condition (20) and rearranging yields:

$$Tr_t^{MP}\left(\frac{1+\tau_t^{C,UFP}}{1+\bar{\tau}^C}\right) + D_t\left(\frac{1+\tau_t^{C,UFP}}{1+\bar{\tau}^C} - 1\right) - T_t^{UFP} = \bar{B}\left(\frac{1+\tau_t^{C,UFP}}{1+\bar{\tau}^C}\right)\left(-r_t^{MP}\right).$$

Multiplying by $\frac{1+\bar{\tau}^C}{1+\tau_t^{C,UFP}}$ yields

$$Tr_t^{MP} + D_t\left(1 - \frac{1+\bar{\tau}^C}{1+\tau_t^{C,UFP}}\right) - T_t^{UFP}\frac{1+\bar{\tau}^C}{1+\tau_t^{C,UFP}} = -r_t^{MP}\bar{B}.$$

This is the same as in the monetary policy experiment if

$$D_t\left(1-\frac{1+\bar{\tau}^C}{1+\tau_t^{C,UFP}}\right)-T_t^{UFP}\frac{1+\bar{\tau}^C}{1+\tau_t^{C,UFP}}=-T_t^{MP}. \tag{41}$$

Using the goods market clearing condition, $Y_t^*=C_t^*$, and the profit equation $D_t^*=Y_t^*-w_t^*L_t^*$, it now holds that (dropping the superscript UFP for the sake of readability):

$$\begin{aligned}
D_t^*\left(\frac{1+\tau_t^C}{1+\bar{\tau}^C}-1\right)*\frac{1+\tau_t^C}{1+\bar{\tau}^C}(\bar{\tau}^C C_t^*+\bar{\tau}^L w_t^* L_t^*)&=\tau_t^C C_t^*+\tau_t^L w_t^* L_t^*\\
\iff D_t^*(\tau_t^C-\bar{\tau}^C)+C_t^*(\bar{\tau}^C-\tau_t^C)&=(\tau_t^L+\bar{\tau}^C\tau_t^L-\bar{\tau}^L-\tau_t^C\bar{\tau}^L)(C_t^*-d_t^*)\\
\iff (\bar{\tau}^C-\tau_t^C-\tau_t^L-\bar{\tau}^C\tau_t^L+\bar{\tau}^L+\tau_t^C\bar{\tau}^L)C_t^*&=(\bar{\tau}^C-\tau_t^C-\tau_t^L-\bar{\tau}^C\tau_t^L+\bar{\tau}^L+\tau_t^C\bar{\tau}^L)d_t^*
\end{aligned}$$

Thus, the government budget constraint is satisfied if:

$$\bar{\tau}^C-\tau_t^C-\tau_t^L-\bar{\tau}^C\tau_t^L+\bar{\tau}^L+\tau_t^C\bar{\tau}^L=0 \tag{42}$$

$$\begin{aligned}
\iff \bar{\tau}^C-\tau_t^C-(1+\bar{\tau}^C)\tau_t^L+(1+\tau_t^C)\bar{\tau}^L&=0\\
\iff 1+\bar{\tau}^C-(1+\tau_t^C)+(1+\tau_t^C)\bar{\tau}^L&=(1+\bar{\tau}^C)\tau_t^L\\
\iff -(1-\bar{\tau}^L)(1+\tau_t^C)&=(\tau_t^C-1)(1+\bar{\tau}^C)
\end{aligned}$$

$$\iff \frac{1-\tau_t^L}{1+\tau_t^C}=\frac{1-\bar{\tau}^L}{1+\bar{\tau}^C}. \tag{43}$$

Which holds given that τ_t^C and τ_t^L are set according to (21).

Consistency with optimal behavior of firms. Given the same households' behavior $\{c_{h,t}^*,l_{h,t}^*\}_{t=0}^{\infty}$ in both policy cases, the firms also face the same demand for goods and the same supply of labor. Hence, if $\{w_t^*,D_t^*,\pi_t^*,\tilde{p}_t{}^*/P_t^*\}_{t=0}^{\infty}$, are equilibrium paths in the monetary policy experiment, they are also equilibrium paths in the UFP case.

Market clearing conditions. From individual behavior x_h^{MP} and x_h^{UFP}, it follows that the sequence of distributions in the HANK-UFP case is $\left\{\Gamma_t^{UFP}(\frac{1+\tau_{t-1}^C}{1+\bar{\tau}^C}b,z)\right\}_{t=0}^{\infty}=\left\{\Gamma_t^{MP}(b,z)\right\}_{t=0}^{\infty}$. That is, if the asset position is adjusted by the consumption value, the distributions are equivalent. Hence, if the asset market clears in the monetary policy experiment, aggregate savings are $B_{t+1}^{d,UFP}=\frac{1+\tau_t^C}{1+\bar{\tau}^C}\bar{B}$ with HANK-UFP which is equal to the supply of government bonds in the HANK-UFP case.

Given the same behavior of firms and the same consumption and labor supply of households, X^{UFP} also clears all other markets if X^{MP} clears all other markets. Thus, we have proven that UFP set according to Proposition 1 yields the same allocation as in the monetary policy experiment which implies that UFP and monetary policy are perfect substitutes in HANK.

B Proof of Lemma 1

Policy-exposure to monetary policy. We derive each household's policy-exposure in the monetary policy experiment, $\Xi^{MP}_{h,t}$, which is defined as the net excess in resources for each household given that only policy variables change. To this end, without loss of generality, we fix household j and consider her budget constraint (equation (2)) in some period t where the real interest rate is r^{MP}_t and consumption taxes and labor taxes as well as the government debt level are at their steady state levels $\bar{\tau}^C, \bar{\tau}^C, \bar{B}$, respectively. Consistent with our definition of the policy-induced redistribution, we set $(c_{j,t}, l_{j,t}, \frac{b_{j,t}}{1+\tau^C_t}, d_t, w_t) = (\bar{c}_j, \bar{l}_j, \frac{\bar{b}'_j}{1+\bar{\tau}}, \bar{d}, \bar{w})$. This yields the following expression for her policy-induced redistribution:

$$\Xi^{MP}_{j,t} = -\big(1+\bar{\tau}^C\big)\bar{c}_j - \bar{b}'_j + (1+r^{MP}_t)\bar{b}_j + \big(1-\bar{\tau}^L\big)\bar{w}z_{j,t}\bar{l}_j + \bar{D} + \tilde{Tr}^{MP}_t. \tag{44}$$

Solving the government budget constraint with the interest rate at period t (but with constant behavior of the agents) gives $\tilde{Tr}^{MP}_t = -r^{MP}_t\bar{B} + \bar{T}$. This can be interpreted as the policy-induced partial equilibrium transfer. Hence, $\Xi^{MP}_{j,t}$ is only affected by the changed return on savings and the direct effect of the real interest rate on transfers. Inserting $\tilde{Tr}_t$ in equation (44) and using the steady state budget constraint of household j yields:

$$\begin{aligned}\Xi^{MP}_{j,t} &= \big(-r^{MP}_t\big)\bar{B} + \bar{T} - \big(-\bar{r}\big)\bar{B} - \bar{T} + \bar{b}_j\big(r^{MP}_t - \bar{r}\big)\\ &= \bar{B}\big(\bar{r} - r^{MP}_t\big) - \bar{b}_j\big(\bar{r} - r^{MP}_t\big).\end{aligned}$$

Policy-exposure to HANK-UFP. We now derive the policy-exposure with HANK-UFP, $\Xi^{UFP}_{h,t}$. To this end, without loss of generality, we fix household j and consider her budget constraint (equation (2)) in some period t, where the real interest rate is at its steady state level $\bar{r}$ and consumption taxes and labor taxes as well as the government debt level are set according to Proposition 1. Consistent with our definition of the policy-induced redistribution, we set $(c_{j,t}, l_{j,t}, \frac{b_{j,t+1}}{1+\tau^C_t}, d_t, w_t) = (\bar{c}_j, \bar{l}_j, \frac{\bar{b}'_j}{1+\bar{\tau}}, \bar{d}, \bar{w})$. This gives the following expression for her policy-induced redistribution:

$$\Xi^{UFP}_{j,t} = -\big(1+\tau^{C,UFP}_t\big)\bar{c}_j - \frac{1+\tau^{C,UFP}_t}{1+\bar{\tau}^{C,UFP}}\bar{b}'_j + (1+\bar{r})\frac{1+\tau^C_{t-1}}{1+\bar{\tau}^C}\bar{b}_j + \big(1-\tau^{L,UFP}_t\big)\bar{w}z_{j,t}\bar{l}_j + \bar{D} + \tilde{Tr}^{UFP}_t.$$

Dividing by gross consumption taxes, inserting the budget constraint in the original steady state, and using condition (21) yields:

$$\frac{\Xi_{j,t}^{UFP}}{1+\tau_t^{C,UFP}} = -(1+\bar{r})\frac{\bar{b}_j}{(1+\bar{\tau}^C)} + (1+\bar{r})\frac{\frac{1+\tau_{t-1}^{C,UFP}}{1+\bar{\tau}^C}\bar{b}_j}{(1+\tau_t^{C,UFP})} + \frac{\bar{D}}{1+\tau_t^{C,UFP}} - \frac{\bar{D}}{1+\bar{\tau}^C} + \frac{\tilde{Tr}_t^{UFP}}{1+\tau_t^{C,UFP}} - \frac{\bar{Tr}}{1+\bar{\tau}^C}.$$

Rearranging and using condition (20) yields:

$$\frac{\Xi_{j,t}^{UFP}}{1+\tau_t^{C,UFP}} = \frac{\bar{b}_j}{(1+\bar{\tau}^C)}\left(r_t^{MP}-\bar{r}\right) + D\left(\frac{\bar{1}}{1+\tau_t^{C,UFP}} - \frac{\bar{1}}{1+\bar{\tau}^C}\right) + \frac{\tilde{Tr}_t^{UFP}}{1+\tau_t^{C,UFP}} - \frac{\bar{Tr}}{1+\bar{\tau}^C}.$$

Solving the government budget constraint and using condition (22) gives the policy-induced transfer in the HANK-UFP case, $\tilde{Tr}_t^{UFP} = \bar{B}\frac{1+\tau_t^{C,UFP}}{1+\bar{\tau}^C} - (1+\bar{r})\frac{1+\tau_{t-1}^{C,UFP}}{1+\bar{\tau}^C}\bar{B} + \tilde{T}_t^{UFP}$, where $\tilde{T}_t^{UFP}$ is the policy-induced partial equilibrium tax income, i.e., the tax income with steady state consumption and labor supply but with HANK-UFP tax rates. Inserting this and the steady state transfer as well as rearranging yields:

$$\begin{aligned}\frac{\Xi_{j,t}^{UFP}}{1+\tau_t^{C,UFP}} = {} & \frac{\bar{b}_j}{(1+\bar{\tau}^C)}\left(r_t^{MP}-\bar{r}\right) + D\left(\frac{1}{1+\tau_t^{C,UFP}} - \frac{1}{1+\bar{\tau}^C}\right) \\ & -\frac{\bar{B}}{(1+\bar{\tau}^C)}\left(r_t^{MP}-\bar{r}\right) + \frac{\tilde{T}_t^{UFP}}{1+\tau_t^{C,UFP}} - \frac{\bar{T}}{1+\bar{\tau}^C}.\end{aligned}$$

Multiplying with $1+\bar{\tau}^C$ and further rearranging yields:

$$\begin{aligned}\Xi_{j,t}^{UFP}\frac{1+\bar{\tau}^C}{1+\tau_t^{C,UFP}} = {} & \bar{B}\left(\bar{r}-r_t^{MP}\right) - \bar{b}_j\left(\bar{r}-r_t^{MP}\right) \\ & +D\left(\frac{1+\bar{\tau}^C}{1+\tau_t^{C,UFP}} - 1\right) + \frac{1+\bar{\tau}^C}{1+\tau_t^{C,UFP}}\tilde{T}_t^{UFP} - \bar{T}.\end{aligned}$$

Given condition (21), the last three terms add up to zero as shown in Appendix A starting from equation (41).[16] Hence,

$$\Xi_{j,t}^{UFP} = \frac{1+\tau_t^{C,UFP}}{1+\bar{\tau}^C}\Xi_{j,t}^{MP}.$$

C Extension to Sticky Wages

We here follow Auclert et al. (2018). Labor hours are determined by union labor demand and we assume that every worker h provides l_{hkt} hours of work to each continuum of unions

[16] Too see this, replace $C_t^*, L_t^*, w_t^*, D_t^*$ in the equations following equation (41) with their steady state values.

indexed $k \in [0,1]$. Total labor effort for person h is therefore $l_{ht} \equiv \int_k l_{hkt} dk$. Each union k aggregates efficient units of work into a union-specific task $L_{ht} = \int z_{ht} l_{hkt} dh$. A competitive labor packer then packages these tasks into aggregate employment services using the technology with constant elasticity of substitution $L_t = \left(\int_k L_{kt}^{\frac{\epsilon-1}{\epsilon}} dk \right)^{\frac{\epsilon}{\epsilon-1}}$ and sells these services to final goods firms at price w_t.

There is a quadratic utility cost of adjusting nominal wage W_{kt} set by union k through an extra additive disutility term $\frac{\nu}{2} \int_k \left(\frac{w_{kt}}{w_{kt-1}} - 1 \right)^2 dk$ in household utility (1). In each period t, union k sets a common wage w_{kt} per efficient unit for each of its members, and calls upon its members to supply hours according to a uniform rule, such that $l_{hkt} = L_{kt}$. The union sets w_{kt} to maximize the average utility of its members given this allocation rule.

In this setup, all unions choose to set the same wage $w_{kt} = w_t$ at time t and all households work the same number of hours, equal to $l_{ht} = L_t$ so efficiency-weighted hours worked $\int z_{it} l_{it} di$ are also equal to aggregate labor demand L_t. Real wages evolve according to $\frac{1+\pi_t^W}{1+\pi_t} = \frac{w_t}{w_{t-1}}$, where π_t^W is the nominal wage inflation which evolves according to an aggregate non-linear wage Phillips curve

$$\pi_t^w (1+\pi_t^w) = \frac{\epsilon_w}{\nu} \int L_t \left(L_t^{\psi} - \frac{\epsilon_w - 1}{\epsilon_w} \frac{1-\tau_t^L}{1+\tau_t^C} w_t z_{ht} c_{ht}^{-\gamma} \right) dh + \beta \pi_{t+1}^w (1+\pi_{t+1}^w),$$

where ϵ_w denotes the elasticity of substitution among unions.

D Extension to Investment

In this section, we present our extension to investment.

D.1 Households

The household problem is the same as the one described in Section (2.1) except that now households face the budget constraints described in equation (31). In the case in which capital and bonds are perfect substitutes, $\omega_{h,t} = 0$.

D.2 Intermediate Good Firms

Intermediate goods are produced by a continuum of intermediate good firms in monopolistically competitive markets. They now produce according to the following Cobb-Douglas production function:

$$y_{j,t} = n_{j,t}^{1-\alpha} k_{j,t}^{\alpha},$$

where $0 < \alpha < 1$, $n_{j,t}$ is labor services, and $k_{j,t}$ is capital services rented in perfectly competitive factor markets. We assume that the intermediate good firms are subject to the same Calvo-pricing as in our baseline model.

D.3 Fiscal Policy

The government's budget constraint is given by equation (30). In addition, we set $\bar{\tau}^F = 0$. In the case in which bonds and capital are perfect substitutes, $\Omega_t = 0 \; \forall t$.

D.4 Equilibrium

Our definition of an equilibrium of this extended economy is analogous to Section 2.4.

D.5 Proof of equivalence when bonds and capital are perfect substitutes

We now prove that conditions (28) and (26) together with conditions (20) and (21) are sufficient conditions for HANK-UFP to generate the same allocation as monetary policy in our HANK model with capital. As in Appendix A, the same allocation implies that households' savings in the HANK-UFP case and in the monetary policy case are the same in consumption value terms.

Satisfying the firms' first-order condition. Given that capital subsidies are set according to condition (26), the firms' first-order condition (25) is the same with HANK-UFP and monetary policy. Note that this also implies that firms' costs are the same since they pay the same net rental rate for the capital they use in the production.

Satisfying household's first-order conditions. Both HANK-UFP and monetary policy generate the same effects on households' first-order conditions since the logic of Appendix A carries over. Hence, if $x_j^{MP} = \{c_{j,t}^*, l_{j,t}^*\}_{t=0}^{\infty}$, satisfies her sequences of Euler equations and labor leisure equations in the monetary policy experiment, $x_j^{UFP} = \{c_{j,t}^*, l_{j,t}^*\}_{t=0}^{\infty}$ does so in the HANK-UFP case.

Satisfying household's budget constraint. The budget constraint of households is now given by equation (27). Denote by $a_{h,t} = b_{h,t} + k_{h,t}$ the total savings of a household. As in Appendix A, we show that if $x_j^{MP} = \{a^*_{j,t+1}, c^*_{j,t}, l^*_{j,t}\}_{t=0}^{\infty}$ satisfies the sequence of budget constraints of a given household j in the monetary policy experiment, $x_j^{UFP} = \{\frac{1+\tau_t}{1+\bar{\tau}} a^*_{j,t+1}, c^*_{j,t}, l^*_{j,t}\}_{t=0}^{\infty}$ does so with HANK-UFP. First note that the labor income term and the lump-sum income term do not change compared to our baseline result given that the transfers resulting as a residual from the government budget constraint generate the transfer path given by equation (23) which we show below to be the case.

The asset income term, however, is now different. It is the same iff:

$$\frac{1 + r_t^{MP}}{1 + \bar{\tau}^C} a^*_{j,t} - \frac{a^*_{j,t+1}}{1 + \bar{\tau}^C} = \frac{1 + \bar{r}}{1 + \tau_t^{C,UFP}} a_{j,t}^{UFP} - \frac{a_{j,t+1}^{UFP}}{1 + \tau_t^{C,UFP}}. \tag{45}$$

Hence, we obtain equivalence iff:

$$a^*_{j,t} \frac{1 + r_t^{MP}}{1 + \bar{\tau}^C} - \frac{a^*_{j,t+1}}{1 + \bar{\tau}^C} = \frac{1 + \bar{r}}{1 + \tau_t^{C,UFP}} \frac{1 + \tau_{t-1}^{C,UFP}}{1 + \bar{\tau}^C} a^*_{j,t} - \frac{1 + \tau_t^{C,UFP}}{1 + \bar{\tau}^C} \frac{a^*_{j,t+1}}{1 + \tau_t^{C,UFP}},$$

which holds given Condition (20).

Note that $a_{j,t+1}^{UFP} = \frac{1+\tau_t^{C,UFP}}{1+\bar{\tau}^C} a^*_{j,t+1}$ is feasible for every j given condition (28) which can be seen by using condition (20) in condition (28).

Equivalence in government budget constraint. We now show that the residual transfers in the HANK-UFP case indeed follow (23). We do so by plugging (23) into the government budget constraint in the HANK-UFP case and show that it holds. To make the following calculations more readable, we now denote $K_t^{MP} = K_t^{UFP} = K_t$, $\tau_t^{C,UFP} = \tau_t^C$, $\tau_t^{L,UFP} = \tau_t^L$, $\tau_t^{F,UFP} = \tau_t^F$, $B_t^{UFP} = B_t$, and $r_t^{MP} = r_t$.

The government budget constraint in the HANK-UFP case is given by

$$Tr_t^{UFP} + (1 + \bar{r})B_t + \tau_t^F K_t = B_{t+1} + T_t^{UFP}. \tag{46}$$

We first plug the transfer path in the HANK-UFP case into (46), given by:

$$Tr_t^{UFP} = \frac{1 + \tau_t^C}{1 + \bar{\tau}^C} Tr_t^{MP} + D_t \left(\frac{1 + \tau_t^C}{1 + \bar{\tau}^C} - 1 \right). \tag{47}$$

We then use the government budget constraint in the monetary policy case given by $Tr_t^{MP} = -(1+r_t)\bar{B} + \bar{B} + T_t^{MP} = -r\bar{B} + T_t^{MP}$, $T_t^{MP} = \bar{\tau}^C(Y_t - K_{t+1} + (1-\delta)K_t) + \bar{\tau}^L(Y_t - (r_t + \delta)K_t - D_t)$, $T_t^{UFP} = {\tau_t}^C(Y_t - K_{t+1} + (1-\delta)K_t) + \tau_t^L(Y_t - (r_t + \delta)K_t - D_t)$, $\tau_t^F = \bar{r} - r_t$ (assuming that

$\bar{\tau}^F = 0$ and using that $r_t = r_t^k$) and rewrite condition (28) by using condition (20) into $B_{t+1} = \frac{(1+\tau_{t+1}^C)(1+r_{t+1})}{(1+\bar{\tau}^C)(1+\bar{r})}\bar{B} + \frac{(1+\tau_{t+1}^C)(1+r_{t+1})}{(1+\bar{\tau}^C)(1+\bar{r})}K_{t+1} - K_{t+1}$ which yields

$$\begin{aligned}
\frac{1+\tau_t^C}{1+\bar{\tau}^C}\Big(-r_t\bar{B} + \bar{\tau}^C(Y_t - K_{t+1} + (1-\delta)K_t) + \bar{\tau}^L(Y_t - (r_t+\delta)K_t - D_t)\Big) + D_t\Big(\frac{1+\tau_t^C}{1+\bar{\tau}^C} - 1\Big) + \\
(\bar{r} - r_t)K_t + \frac{(1+\tau_t^C)(1+r_t)}{(1+\bar{\tau}^C)}\bar{B} + \frac{(1+\tau_t^C)(1+r_t)}{(1+\bar{\tau}^C)}K_t - (1+\bar{r})K_t \\
= \frac{(1+\tau_{t+1}^C)(1+r_{t+1})}{(1+\bar{\tau}^C)(1+\bar{r})}\bar{B} + \frac{(1+\tau_{t+1}^C)(1+r_{t+1})}{(1+\bar{\tau}^C)(1+\bar{r})}K_{t+1} - K_{t+1} \\
+ {\tau_t}^C(Y_t - K_{t+1} + (1-\delta)K_t) + \tau_t^L(Y_t - (r_t+\delta)K_t - D_t)
\end{aligned} \tag{48}$$

We will show that this holds by collecting appropriate terms one by one and show that these terms drop out. We start by collecting all terms involving dividends and show that these cancel out:

$$\begin{aligned}
\Big(-\frac{1+\tau_t^C}{1+\bar{\tau}^C}\bar{\tau}^L + \frac{1+\tau_t^C}{1+\bar{\tau}^C} - 1 + \tau_t^L\Big)D_t = 0 \\
\iff (-\bar{\tau}^L - \bar{\tau}^L\tau_t^C + 1 + \tau_t^C - 1 - \bar{\tau}^C + {\tau_t}^L + \bar{\tau}^C\tau_t^L)D_t = 0 \\
\iff \underbrace{(-\bar{\tau}^L - \bar{\tau}^C - \bar{\tau}^L\tau_t^C + \tau_t^C + {\tau_t}^L + \bar{\tau}^C\tau_t^L)}_{=0, \text{ see } (42)}D_t = 0
\end{aligned}$$

Next, we show that all the terms involving $\bar{B}$ are 0.

$$\begin{aligned}
-r_t\frac{1+\tau_t^C}{1+\bar{\tau}^C}\bar{B} + \frac{1+\tau_t^C}{1+\bar{\tau}^C}(1+r_t)\bar{B} - \frac{1+\tau_{t+1}^C}{1+\bar{\tau}^C}\frac{1+r_{t+1}}{1+\bar{r}}\bar{B} = 0 \\
\iff \frac{1+\tau_t^C}{1+\bar{\tau}^C}\bar{B} - \frac{1+\tau_t^C}{1+\bar{\tau}^C}\frac{1+\bar{r}}{1+\bar{r}}\bar{B} = 0
\end{aligned}$$

Hence, equation (48) can then be simplified to:

$$\begin{aligned}
\frac{1+\tau_t^C}{1+\bar{\tau}^C}\bar{\tau}^C(Y_t - K_{t+1} + (1-\delta)K_t) + (\bar{r} - r_t)K_t + \\
+ \frac{(1+\tau_t^C)(1+r_t)}{(1+\bar{\tau}^C)}K_t - (1+\bar{r})K_t \\
= \frac{(1+\tau_{t+1}^C)(1+r_{t+1})}{(1+\bar{\tau}^C)(1+\bar{r})}K_{t+1} - K_{t+1} - \frac{1+\tau_t^C}{1+\bar{\tau}^C}\bar{\tau}^L(Y_t - (r_t+\delta)K_t) + \\
{\tau_t}^C(Y_t - K_{t+1} + (1-\delta)K_t) + \tau_t^L(Y_t - (r_t+\delta)K_t)
\end{aligned} \tag{49}$$

Rearranging yields

$$\begin{aligned}
&\left(\frac{1+\tau_t^C}{1+\bar{\tau}^C}\bar{\tau}^C-\tau_t^C\right)(Y_t-K_{t+1}+(1-\delta)K_t)\\
+&\left(\frac{1+\tau_t^C}{1+\bar{\tau}^C}\bar{\tau}^L-\tau_t^L\right)(Y_t-(r_t+\delta)K_t)=-(\bar{r}-r_t)K_t-\frac{(1+\tau_t^C)(1+r_t)}{(1+\bar{\tau}^C)}K_t+(1+\bar{r})K_t\\
&+\frac{1+\tau_t^C}{1+\bar{\tau}^C}K_{t+1}-K_{t+1}
\end{aligned} \tag{50}$$

We next show that all the terms in (50) involving Y_t are 0:

$$\begin{aligned}
&\frac{1+\tau_t^C}{1+\bar{\tau}^C}\bar{\tau}^C Y_t-\tau_t^C Y_t+\frac{1+\tau_t^C}{1+\bar{\tau}^C}\bar{\tau}^L Y_t-\tau_t^L Y_t=0\\
\iff &(1+\tau_t^C)\bar{\tau}^C Y_t-(1+\bar{\tau}^C)\tau_t^C Y_t+(1+\tau_t^C)\bar{\tau}^L Y_t-(1+\bar{\tau}^C)\tau_t^L Y_t=0\\
\iff &Y_t(\bar{\tau}^C+\bar{\tau}^C\tau_t^C-\tau_t^C-\bar{\tau}^C\tau_t^C+\bar{\tau}^L+\tau_t^C\bar{\tau}^L-\tau_t^L-\bar{\tau}^C\tau_t^L)=0\\
\iff &Y_t(\underbrace{\bar{\tau}^C-\tau_t^C-\tau_t^L-\bar{\tau}^C\tau_t^L+\bar{\tau}^L+\tau_t^C\bar{\tau}^L}_{=0,\text{ see (42)}})=0
\end{aligned}$$

We next show that all the terms in (50) involving K_{t+1} are 0:

$$\begin{aligned}
&\left(\frac{1+\tau_t^C}{1+\bar{\tau}^C}\bar{\tau}^C-\tau_t^C\right)K_{t+1}+\frac{1+\tau_t^C}{1+\bar{\tau}^C}K_{t+1}-K_{t+1}=0\\
\iff &\frac{1+\tau_t^C}{1+\bar{\tau}^C}\bar{\tau}^C K_{t+1}-\tau_t^C K_{t+1}+\frac{1+\tau_t^C}{1+\bar{\tau}^C}K_{t+1}-K_{t+1}=0\\
\iff &K_{t+1}\left(\frac{1+\tau_t^C}{1+\bar{\tau}^C}\bar{\tau}^C-(\tau_t^C+1)+\frac{1+\tau_t^C}{1+\bar{\tau}^C}\right)=0\\
\iff &K_{t+1}(\bar{\tau}^C+\tau_t^C\bar{\tau}^C-\tau_t^C-1-\bar{\tau}^C\tau_t^C-\bar{\tau}^C+1+\tau_t^C)=0
\end{aligned}$$

We next show that all the terms in (50) involving δK_t are 0:

$$
\begin{aligned}
-\left(\frac{1+\tau_t^C}{1+\bar{\tau}^C}\bar{\tau}^C-\tau_t^C\right)\delta K_t-\left(\frac{1+\tau_t^C}{1+\bar{\tau}^C}\bar{\tau}^L-\tau_t^L\right)\delta K_t&=0\\
\iff \delta K_t\left(-\frac{1+\tau_t^C}{1+\bar{\tau}^C}\bar{\tau}^C+\tau_t^C-\frac{1+\tau_t^C}{1+\bar{\tau}^C}\bar{\tau}^L+\tau_t^L\right)&=0\\
\iff \delta K_t(-(1+\tau_t^C)\bar{\tau}^C+\tau_t^C+\bar{\tau}^C\tau_t^C-(1+\tau_t^C)\bar{\tau}^L+\tau_t^L\bar{\tau}^C+\tau_t^L)&=0\\
\iff \delta K_t(-\bar{\tau}^C-\tau_t^C\bar{\tau}^C+\tau_t^C+\bar{\tau}^C\tau_t^C-\bar{\tau}^L-\tau_t^C\bar{\tau}^L+\tau_t^L\bar{\tau}^C+\tau_t^L)&=0\\
\iff \delta K_t(\underbrace{\tau_t^C+\tau_t^L-\bar{\tau}^C-\bar{\tau}^L-\tau_t^C\bar{\tau}^L+\tau_t^L\bar{\tau}^C}_{=0\text{, see (42)}})&=0
\end{aligned}
$$

Hence, equation (50) can now be simplified to:

$$
\left(\frac{1+\tau_t^C}{1+\bar{\tau}^C}\bar{\tau}^C-\tau_t^C\right)K_t-\left(\frac{1+\tau_t^C}{1+\bar{\tau}^C}\bar{\tau}^L-\tau_t^L\right)r_tK_t-r_tK_t-K_t+\frac{1+\tau_t^C}{1+\bar{\tau}^C}(1+r_t)K_t=0 \quad (51)
$$

We next show that all the terms in (51) involving r_tK_t are 0:

$$
\begin{aligned}
-\left(\frac{1+\tau_t^C}{1+\bar{\tau}^C}\bar{\tau}^L-\tau_t^L\right)r_tK_t-r_tK_t+\frac{1+\tau_t^C}{1+\bar{\tau}^C}r_tK_t&=0\\
\iff r_tK_t\left(-\frac{1+\tau_t^C}{1+\bar{\tau}^C}\bar{\tau}^L+\tau_t^L-1+\frac{1+\tau_t^C}{1+\bar{\tau}^C}\right)&=0\\
\iff r_tK_t(-\bar{\tau}^L-\tau_t^C\bar{\tau}^L+\tau_t^C+\bar{\tau}^C\tau_t^L-1-\bar{\tau}^C+1+\tau_t^C)&=0\\
\iff r_tK_t(\underbrace{\tau_t^C+\tau_t^L-\bar{\tau}^L-\bar{\tau}^C-\tau_t^C\bar{\tau}^L+\bar{\tau}^C\tau_t^L}_{=0\text{, see (42)}})&=0
\end{aligned}
$$

We finally show that all the terms in (51) involving K_t are 0:

$$
\begin{aligned}
\left(\frac{1+\tau_t^C}{1+\bar{\tau}^C}\bar{\tau}^C-\tau_t^C-1+\frac{1+\tau_t^C}{1+\bar{\tau}^C}\right)K_t&=0\\
K_t(\bar{\tau}^C+\bar{\tau}^C\tau_t^C-\tau_t^C-\bar{\tau}^C\tau_t^C-1-\bar{\tau}^C+1+\tau_t^C)&=0
\end{aligned}
$$

D.6 Proof of equivalence when bonds and capital are imperfect substitutes

When allowing for $r_t \neq r_t^k$, where r_t^k is the return on capital net of depreciation, the budget constraint of a given household j is now given by equation (31). We now denote $a_{h,t+1} = k_{h,t+1} + \omega_{h,t+1}$ as the total illiquid asset holdings of a household. To make the following calculations more readable, we now denote $K_t^{MP} = K_t^{UFP} = K_t$, $\tau_t^{C,UFP} = \tau_t^C$, $\tau_t^{L,UFP} = \tau_t^L$, $\tau_t^{F,UFP} = \tau_t^F$, $B_t^{UFP} = B_t$, and $r_t^{MP} = r_t$. Assuming equivalence in illiquid asset income, we obtain an expression for $1 + r_t^{k,UFP}$:

$$\begin{aligned}
\frac{1+r_t^{k,MP}}{1+\bar{\tau}^C} a_{j,t}^* - \frac{a_{j,t+1}^*}{1+\bar{\tau}^C} &= \frac{1+r_t^{k,UFP}}{1+\tau_t^C} \frac{1+\tau_{t-1}^C}{1+\bar{\tau}^C} a_{j,t}^* - \frac{\frac{1+\tau_t^C}{1+\bar{\tau}^C} a_{j,t+1}^*}{1+\tau_t^C} \\
\iff \frac{1+r_t^{k,MP}}{1+\bar{\tau}^C} &= \frac{1+r_t^{k,UFP}}{1+\tau_t^C} \frac{1+\tau_{t-1}^C}{1+\bar{\tau}^C} \\
\iff 1+r_t^{k,MP} &= \left(1+r_t^{k,UFP}\right) \frac{1+\tau_{t-1}^C}{1+\tau_t^C} \\
\iff 1+r_t^{k,UFP} &= \left(1+r_t^{k,MP}\right) \frac{1+\bar{r}}{1+r_t}
\end{aligned}$$

From the firms' first-order conditions, we know:

$$\begin{aligned}
1+r_t^{k,MP} &= 1+r_t^{k,UFP} - \tau_t^F \\
\iff \tau_t^F &= 1+r_t^{k,UFP} - \left(1+r_t^{k,MP}\right) \\
\iff \tau_t^F &= \left(1+r_t^{k,MP}\right) \frac{1+\bar{r}}{1+r_t} - \left(1+r_t^{k,MP}\right) \\
\iff \tau_t^F &= \left(1+r_t^{k,MP}\right) \left(\frac{1+\bar{r}}{1+r_t} - 1\right)
\end{aligned}$$

Equivalence in the government budget constraint. With synthetic capital, Ω_t, as a second illiquid asset, the government budget constraint is given by equation (30) and Ω_t is set according to condition (32). Doing the same rearrangements as before, this yields:

$$\frac{1+\tau_t^C}{1+\bar{\tau}^C}\Big(-r_t\bar{B}+\bar{\tau}^C(Y_t-K_{t+1}+(1-\delta)K_t)+\bar{\tau}^L(Y_t-(r_t^{k,MP}+\delta)K_t-D_t)\Big)$$
$$+D_t\Big(\frac{1+\tau_t^C}{1+\bar{\tau}^C}-1\Big)+(1+r_t^{k,MP})\Big(\frac{1+\bar{r}}{1+r_t}-1\Big)K_t+$$
$$(1+\bar{r})\frac{(1+\tau_t^C)(1+r_t)}{(1+\bar{\tau}^C)(1+\bar{r})}\bar{B}+(1+r_t^{k,UFP})K_t\Big(\frac{1+\tau_{t-1}^C}{1+\bar{\tau}^C}-1\Big)=$$
$$\frac{1+\tau_t^C}{1+\bar{\tau}^C}\bar{B}+\Big(\frac{1+\tau_t^C}{1+\bar{\tau}^C}-1\Big)K_{t+1}+\tau_t{}^C(Y_t-K_{t+1}+(1-\delta)K_t)+\tau_t^L(Y_t-(r_t^{k,MP}+\delta)K_t-D_t) \tag{52}$$

Note that all terms except for the K_t and K_{t+1} terms are the same as in Appendix D.5. Hence, all of them cancel out. What remains to be shown, is that the K_t and K_{t+1} terms also cancel out in the model with illiquid capital. We start by showing that the terms in (52) involving K_t are 0:

$$\frac{1+\tau_t^C}{1+\bar{\tau}^C}\bar{\tau}^C(1-\delta)K_t-\frac{1+\tau_t^C}{1+\bar{\tau}^C}\bar{\tau}^L\big(r_t^{k,MP}+\delta\big)K_t+\big(1+r_t^{k,MP}\big)\Big(\frac{1+\bar{r}}{1+r_t}-1\Big)K_t$$
$$+\big(1+r_t^{k,UFP}\big)K_t\Big(\frac{(1+\tau_t^C)(1+r_t)}{(1+\bar{\tau}^C)(1+\bar{r})}-1\Big)-\tau_t^C(1-\delta)K_t+\tau_t^L\big(r_t^{k,MP}+\delta\big)K_t=0 \tag{53}$$

To do so, we first show that only the terms involving δK_t in (53) drop out

$$-\frac{1+\tau_t^C}{1+\bar{\tau}^C}\bar{\tau}^C\delta K_t-\frac{1+\tau_t^C}{1+\bar{\tau}^C}\bar{\tau}^L\delta K_t+\tau_t^C\delta K_t+\tau_t^L\delta K_t=0$$
$$\delta K_t(-\bar{\tau}^C-\bar{\tau}^C\tau_t^C-\bar{\tau}^L+\tau_t^C\bar{\tau}^L+\tau_t^C+\bar{\tau}^C\tau_t^C+\tau_t^L+\bar{\tau}^C\tau_t^L)=0$$
$$\delta K_t(\underbrace{-\bar{\tau}^C-\bar{\tau}^L+\tau_t^C\bar{\tau}^L+\tau_t^C+\tau_t^L+\bar{\tau}^C\tau_t^L}_{=0,\text{ see (42)}})=0$$

We next show that the rest of equation (53) is also zero:

$$\frac{1+\tau_t^C}{1+\bar{\tau}^C}\bar{\tau}^CK_t-\frac{1+\tau_t^C}{1+\bar{\tau}^C}\bar{\tau}^Lr_t^{k,MP}K_t+\big(1+r_t^{k,MP}\big)\frac{1+\bar{r}}{1+r_t}K_t-(1+r_t^{k,MP})K_t$$
$$+\big(1+r_t^{k,MP}\big)\frac{1+\bar{r}}{1+r_t}\frac{(1+\tau_t^C)(1+r_t)}{(1+\bar{\tau}^C)(1+\bar{r})}K_t-\big(1+r_t^{k,MP}\big)\frac{1+\bar{r}}{1+r_t}K_t$$
$$-\tau_t^CK_t+\tau_t^Lr_t^{k,MP}K_t=0$$

Rearranging yields

$$\begin{aligned}(1+r_t^{k,MP})K_t\left(\frac{1+\bar{r}}{1+r_t}-1+\frac{1+\tau_t^C}{1+\bar{\tau}^C}-\frac{1+\bar{r}}{1+r_t}\right)\\+\frac{1+\tau_t^C}{1+\bar{\tau}^C}\bar{\tau}^C K_t-\frac{1+\tau_t^C}{1+\bar{\tau}^C}\bar{\tau}^L r_t^{k,MP}K_t-\tau_t^C K_t+\tau_t^L r_t^{k,MP}K_t=0\end{aligned}$$

We now show first that the following terms are 0:

$$\begin{aligned}\left(\frac{1+\tau_t^C}{1+\bar{\tau}^C}-1\right)K_t+\frac{1+\tau_t^C}{1+\bar{\tau}^C}\bar{\tau}^C K_t-\tau_t^C K_t=0\\\iff (1+\tau_t^C-1-\bar{\tau}^C+\bar{\tau}^C+\tau_t^C\bar{\tau}^C-\tau_t^C-\bar{\tau}^C\tau_t^C)K_t=0\end{aligned}$$

We next show that the following terms are 0:

$$\begin{aligned}r_t^{k,MP}K_t\left(\frac{1+\tau_t^C}{1+\bar{\tau}^C}-1-\frac{1+\tau_t^C}{1+\bar{\tau}^C}\bar{\tau}^L+\tau_t^L\right)=0\\r_t^{k,MP}K_t(1+\tau_t^C-1-\bar{\tau}^C-\bar{\tau}^L-\tau_t^C\bar{\tau}^L+\tau_t^L+\bar{\tau}^C\tau_t^L)=0\\r_t^{k,MP}K_t(\underbrace{\tau_t^C+\tau_t^L-\bar{\tau}^C-\bar{\tau}^L-\tau_t^C\bar{\tau}^L+\bar{\tau}^C\tau_t^L}_{=0,\text{ see (42)}})=0\end{aligned}$$

Finally, we show that all the terms involving K_{t+1} in (52) are 0:

$$\begin{aligned}-\frac{1+\tau_t^C}{1+\bar{\tau}^C}\bar{\tau}^C K_{t+1}-\frac{1+\tau_t^C}{1+\bar{\tau}^C}K_{t+1}+K_{t+1}+\tau_t^C K_{t+1}=0\\K_{t+1}\left(-\frac{1+\tau_t^C}{1+\bar{\tau}^C}\bar{\tau}^C-\frac{1+\tau_t^C}{1+\bar{\tau}^C}+1+\tau_t^C\right)=0\\K_{t+1}(-\bar{\tau}^C-\tau_t^C\bar{\tau}^C-1-\tau_t^C+1+\bar{\tau}^C+\tau_t^C+\bar{\tau}^C\tau_t^C)=0\end{aligned}$$